MINING ASSOCIATION RULES WITH UNCERTAIN ITEM RELATIONSHIPS

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ABSTRACT

In order to enhance the precision in association rule mining, an extension is proposed to capture the uncertain item relationships in the data sets in this paper. Two sources of uncertainty are considered: the degree of individual item importance (multiplicity) and the degree of association among the items (inter-relationship). In many real-world applications, especially in a distributed environment, the data sets are generated and collected from different sources. Thus the inter-relationships among the items can vary and result in uncertain item relationships. The Dempster-Shafer (DS) evidential reasoning theory is applied to generate the association rules with the proposed support and confidence measures under uncertainty. These measures are defined under Shannon-like measure of total uncertainty. A numerical example based on market basket analysis is given with a comparison between our approach and the original association rule mining method.

Keywords: Knowledge Discovery in Databases, Association Rule Mining, Evidence Theory, Uncertainty Reasoning, and Measure of Total Uncertainty.

1. INTRODUCTION

Association rule mining is one of the most widely applied algorithms in knowledge discovery in databases (KDD) or data mining [4][7][15][16]. Originally proposed by [2], its basic idea is to discover important and interesting associations among the data items such that the presence of some items in a transaction will imply the presence of other items in the same transaction. The outputs generated from the association rule mining are some rules, which pass the user-specified minimum support and confidence measures. The association rule mining is popularly applied to the problem of market basket analysis (MBA), where the data set is a collection of transaction records, each containing a list of items that customers purchase in a single transaction.

Many extensions to the original association rule mining [2] have been proposed to deal with some of the original algorithm's drawbacks. Examples include the use of the interestingness measure to prune down the number of rules [3][11], mining the generalized association rules involving hierarchical data set [17] and generalized affinity-based association rule mining [15][16]. Most of these extensions including the original algorithm make an assumption that the data set under consideration is precise or consistent and contain no ambiguity. However, for many real-world applications, the data set is usually far from being perfect. Data sets commonly contain some uncertainty, particularly incompleteness and inconsistency. One example is a distributed information environment, where the data sets are generated and collected from different sources, and each source may have different constraints. This can lead to different inter-relationships among the items, thus imposing on the data set.

As suggested in [5], uncertainty can be classified into the following categories:

- Incompleteness: this is due to the absence of the value.
- Imprecision: this arises from a value that cannot be measured with suitable precision.
- Imperfection: this is due to the lack of information relative to the state of the world, and may be due to subjective errors on the part of some observer.
- Randomness: this arises due to the inability to differentiate among the items.
- Vagueness: this can be considered as a subcategory of imprecision.
- Inconsistency: this describes a situation where a variable has two or more conflicting values.
- Ignorance: this arises due to lack of knowledge.

We consider two sources of uncertainty, which were first proposed and realized in [10]. The first is the degree of individual item importance, or item multiplicity. The item multiplicity can be classified as randomness, since the number of items occurring in one transaction can be different from other transactions. We allow the multiplicity of items to affect the outcome of the association rule mining process. The second is the degree of inter-relationships among the items. Degree of inter-relationships can be considered as vagueness and inconsistency, since their values can be set differently due to different data sources. In this paper, such differentiation in the data sets is provided, thus making the association rule mining process more generalized.

To deal with the above uncertainty in the data sets, the Dempster-Shafer (DS) evidential reasoning theory [6][13] is applied in the association rule mining process. Unlike some other approaches in handling uncertainty in data sets such as fuzzy set and possibility theory [18], evidence theory allows us to model and construct the itemsets easily via its *basic probability assignment* (*bpa*) or *mass function* (*m*). Using the *bpa*, the item inter-relationships and multiplicity can be conveniently captured within the framework [10] as shown in the next section. We propose the *support* and *confidence* measures under uncertainty using a Shannon-like measure of total uncertainty so that the degree of total uncertainty hidden in the data sets under consideration is captured.

The rest of the paper is organized as follows. In the next section, we discuss the basic notions in evidence theory such as *belief*, *plausibility*, and *bpa* functions as being applied to the association rule mining process. In Section 3, the Shannon-like measure of total uncertainty based on the *belief* and *plausibility* functions in the evidence theory is presented along with the proposed support and confidence measures for the association rule mining. An example of MBA problem is used to illustrate our framework in Section 2 and Section 3. The paper is concluded in Section 4.

2. APPLYING EVIDENCE THEORY TO ASSOCIATION RULE MINING

Dempster-Shafer (DS) theory [6][13] aims to provide a theory of partial belief. It attempts to overcome the representational deficiencies within the probability (or Bayesian) theory as well as to provide some mechanisms for making inferences from the available evidence. The *frame of discernment* (FOD) Θ is the set of mutually exclusive and exhaustive propositions of interest. Defined on the set of all subsets of Θ is the *basic probability assignment* (*bpa*) or the *mass function* (*m*) that associates with every subset of Θ a degree of belief that lies within the interval [0,1]. The *bpa* function can be mathematically defined as follows.

Definition 1 (Basic Probability Assignment): A real positive function *m* given by $m: 2^{\Theta} \to [0,1]$ is a *basic probability assignment* for the FOD Θ if it satisfies the following conditions:

1. $m(\emptyset) = 0$.
2. $\sum_{A \subseteq \Theta} m(A) = 1$.

The quantity *m(A)* is called the basic probability number (or value) of *A*.

The propositions in the FOD Θ that possess non-zero *bpa*s are called the *focal elements* of Θ and are denoted by

$$F(\Theta) = \{ A \subseteq \Theta \mid m(A) > 0 \}.$$

For a given FOD Θ, the triplet $\{\Theta, F, m\}$ is referred to as its *body of evidence* (BOE).

The quantity $m(A)$ measures the support assigned to proposition *A* only and to no smaller subset of *A*. However, the belief assigned to any proposition is also committed to any other proposition it implies. In other words, the belief *A* must take into account the supports for all proper subsets of *A* as well.

Definition 2 (Belief): Given a BOE $\{\Theta, F, m\}$, the belief assigned to $A \subseteq \Theta$ is $Bel: 2^{\Theta} \to [0,1]$, where

$$Bel(A) = \sum_{B \subseteq A} m(B).$$

Thus, $Bel(A)$ represents the total support that can move into *A* without any ambiguity.

Definition 3 (Doubt): Given a BOE $\{\Theta, F, m\}$, the doubt regarding $A \subseteq \Theta$ is $Dou: 2^{\Theta} \to [0,1]$, where

$$Dou(A) = Bel(\overline{A}).$$

The doubt can also be visualized as the collection of floating masses that cannot move into *A* with the acquisition of new information. This can be represented as,

$$Bel(\overline{A}) = \sum_{B \cap A = f} m(B).$$

Definition 4 (Plausibility): Given a BOE $\{\Theta, F, m\}$, the plausibility of $A \subseteq \Theta$ is $Pl: 2^{\Theta} \to [0,1]$, where

$$Pl(A) = 1 - Dou(A) = 1 - Bel(\overline{A}).$$

$Pl(A)$ indicates the extent to which one fails to doubt *A*, i.e., the extent to which one finds *A* to be plausible. The plausibility function can also be rewritten as follows:

$$Pl(A) = \sum_{B \cap A \neq f} m(B).$$

Note that $Pl(A) \geq Bel(A)$, for any $A \subseteq \Theta$.

The definition of the *uncertainty* interval associated with A is denoted by $Un(A)$ as shown below:

$$Un(A) = [Bel(A), Pl(A)].$$

Applications of DS theory notions in association rule mining appear in [10]. For our purpose, we proceed as follows.

We consider a set of items, $I = \{i_1, i_2, \ldots, i_n\}$, where n is the number of items. Then I can be modeled as an FOD in the evidence theory. We also define a set of inter-relationships on I (denoted by r) by using the *basic probability assignment* function. That is, $r: 2^I \rightarrow [0,1]$ is a *bpa* for the FOD I which satisfies the following two conditions.

1. $r(\varnothing) = 0$.
2. $\sum_{A \subseteq I} r(A) = 1$.

Note that the *bpa* function r captures the importance of inter-relationships among the items as determined by the incoming source of the data set. Therefore, r can vary depending on the information or data source.

As an example, we consider a MBA problem. For simplicity, we assume the same FOD, i.e., the same set of items for all data sets. Suppose we have transactions generated from two different sources: Store 1 and Store 2. Under the same FOD assumption, $I_1 = I_2 = \{$dairy, snack, toiletry$\}$. Table 1 shows the sets of inter-relationships among the items for both stores. As can be seen from Table 1, the values for each itemset can vary between Store 1 and Store 2, thus resulting in uncertain inter-relationships among items as mentioned earlier.

Itemsets	**Store 1**	**Store 2**
r (dairy)	0.1	0.2
r (snack)	0.1	0.2
r (toiletry)	0.3	0.2
R (dairy, snack)	0.1	0.1
r (dairy,toiletry)	0.2	0.1
r (snack,toiletry)	0.2	0.1
r (dairy,snack,toiletry)	0.0	0.1
Sum	1.0	1.0

Table 1: The sets of inter-relationships for Store 1 and Store 2.

Suppose the data set from the two stores is as shown in Table 2. Each transaction in a single row contains the amounts of items that a customer purchased. For example, in the first transaction, a customer bought two dairy items and five snack items from Store 1.

Next we apply the r-value set from Table 1 to the transactions in Table 2, thus generating another set of *bpa*s (denoted by m) at the transaction level. Based on the original association rule mining approach [2], the *support* value for a multiple-itemset is determined from the co-occurrence of all items in the itemset. For example, from Transaction 1 in Table 2, the *support* value for {dairy, snack} is equal to *1*, since both dairy and toiletry occurred in the transaction; while the *support* value for {dairy, toiletry} is equal to *0*, since dairy and toiletry did not co-occur in the transaction.

In order to comply with the original association rule mining and to attain the item multiplicity, we propose the use of the minimum strategy to select the representative item whose value is the smallest (minimum) in the itemset. Take Transaction 1 in Table 2 as an example. The *support* value for {dairy, snack} itemset is equal to *2*, since the value of the dairy item is *2*, which is smaller than the value *5* of the snack item, and the *support* value for {dairy, toiletry} itemset is equal to *0* since the value *0* of the toiletry item is the minimum value. Using this minimum strategy, our approach complies with the original association rule mining when one of the items in the itemset is equal to zero, and it captures the multiplicity of the itemsets as well.

Using the minimum strategy described above, the *bpa* process at the transaction level involves the multiplication of the r-value with the representative item value (or amount) of each itemset. Therefore, the degree of item importance (multiplicity) is considered at this step. Then the m-value set is normalized by dividing by the sum of all m values. The normalization process is required to satisfy the second condition of Definition 1.

Transaction No.	**Store**	**dairy**	**snack**	**toiletry**
1	1	2	5	0
2	1	2	1	1
3	1	0	1	1
4	1	1	1	1
5	1	0	0	3
6	1	2	3	0
7	2	1	1	2
8	2	3	2	0
9	2	0	2	3
10	2	1	2	4
11	2	2	0	1

Table 2: Data set collected from two different stores.

The following example is to show how to generate the *bpa* set for a transaction at the transaction level from the *bpa* sets of the inter-relationships (as shown in Table 1). We denote $min(i_1, i_2, \ldots, i_n)$ as the minimum function whose result is the mimimum value among the itemset

$\{i_1, i_2, \ldots, i_n\}$, where n is the number of items in the itemset.

Example 1: Consider Transaction 1 from Table 2. The set of *bpa* values for this transaction can be calculated using the set of inter-relationships from Store 1 as shown below.

m(dairy) = min(dairy) × 0.1 = 2 × 0.1 = 0.2;
m(snack) = min(snack) × 0.1 = 5 × 0.1 = 0.5;
m(toiletry) = min(toiletry) × 0.3 = 0 × 0.3 = 0;
m(dairy,snack) = min(dairy,snack) × 0.1 = 2 × 0.1 = 0.2;
m(dairy,toiletry) = min(dairy,toiletry) × 0.2 = 0 × 0.2 =0;
m(snack,toiletry) = min(snack,toiletry) × 0.2 = 0 × 0.2 =0;
m(dairy,snack,toiletry) = min(dairy,snack,toiletry) × 0 = 0.

Then, we normalize the above set of m-value by dividing by the sum of m-value. Note that the sum of the m-value set is 0.9. The final result of m-value is shown below.

m(dairy) = 0.22;
m(snack) = 0.56;
m(toiletry) = 0;
m(dairy,snack) = 0.22;
m(dairy,toiletry) = 0;
m(snack,toiletry) = 0;
m(dairy,snack,toiletry) = 0.

Next, the set of uncertainty intervals [*Bel(x)*, *Pl(x)*], where x is an itemset, can be constructed from the above m-value set using the *belief* and *plausibility* functions from Definitions 2 and 4, respectively. To comply with the original association rule mining, we only consider [*Bel(x)*, *Pl(x)*] of the itemset whose m-value is greater than zero. We refer to this as the restricted uncertainty interval, which can be defined as follows.

$$\hat{Un}(x) = \begin{cases} [0,0], & m(x) = 0 \\ [Bel(x), Pl(x)], & otherwise \end{cases}$$

Using the restricted uncertainty interval, the calculations from the above m-value set are shown below.

$\hat{Un}$ (dairy) = [0.22, 0.44] ;

$\hat{Un}$ (snack) = [0.56, 0.78] ;

$\hat{Un}$ (toiletry) = [0, 0] ;

$\hat{Un}$ (dairy,snack) = [1, 1] ;

$\hat{Un}$ (dairy,toiletry) = [0, 0] ;

$\hat{Un}$ (snack,toiletry) = [0, 0] ;

$\hat{Un}$ (dairy,snack,toiletry) = [0, 0].

In the next section, the *support* and *confidence* measures of the association rule mining are defined based on the measure of total uncertainty.

3. SUPPORT AND CONFIDENCE BASED ON THE MEASURES OF TOTAL UNCERTAINTY

The concepts of uncertainty and information are closely related. A measure of the uncertainty, which prevails before the experiment is accomplished, can be considered as a measure of the information expected from an experiment. In this paper, we use the terms "measure of the uncertainty" and "measure of the information" as having interchangeable meanings. One of the most well established methods of measuring the information is the classical Shannon measure (or information entropy) [14]. The original Shannon measure applies to uncertainty formalized in terms of a probability distribution $p = \{p(x) \mid x \in X\}$, where p is defined on a s-algebra of measurable subsets of the FOD.

To measure the amount of uncertainty under the probability distribution p, the following function was proposed.

$$S(p) = -\sum_{x \in X} p(x) \log_2 p(x). \qquad \text{Eq. (1)}$$

The above function S yields the measurement unit in *bit* and also has a well-established axiomatic basis [1][12].

In an attempt to measure the total uncertainty or information derived from the evidence theory, many measures based on the Shannon function have been proposed and are referred to as Shannon-like (*SL*) measures. Comparisons and drawbacks of the previous proposed measures are given in [8] and [9].

In this paper, we consider the *SL* measure of uncertainty suggested in [9]. This *SL* measure adopts the function S from Eq. (1) to define the total uncertainty based on the restricted uncertainty interval $\hat{Un}(x)$ as shown below.

$$SL(x) = -\frac{1}{c}\sum_{x \in X} \begin{matrix}[Bel(\{x\})\log_2 Bel(\{x\}) \\ + Pl(\{x\})\log_2 Pl(\{x\})]\end{matrix}, \qquad \text{Eq. (2)}$$

where $c = \sum_{x \in X}[Bel(\{x\}) + Pl(\{x\})]$, and X is the FOD under consideration. For the uncertainty interval with Bel=0 and Pl=0, we assume that the *SL* value is equal to zero (denoting the minimum value). It can be shown that the maximum value of the above *SL* measure occurs when the *Bel* and *Pl* values of the itemset are equal among all transactions (which corresponds to a uniform distribution over all transactions). Given n transactions, the maximum *SL* value can be derived as shown below.

$$SL_{\max} = -\frac{1}{c}\sum_{n}[\left(\frac{1}{n}\right)\log_2\left(\frac{1}{n}\right) + \left(\frac{1}{n}\right)\log_2\left(\frac{1}{n}\right)],$$

where $c = \sum_{n}[\left(\frac{1}{n}\right) + \left(\frac{1}{n}\right)]$.

Therefore, we have

$$SL_{\max} = -\log_2\left(\frac{1}{n}\right). \quad \text{Eq. (3)}$$

We now give the definitions of the *support* and *confidence* measures for the association rule mining based on the above *SL* measure. These measures are adopted from the initial suggestion given in [10].

$$Support\ (X \Rightarrow Y) = \sum_k SL\ (X, Y), \quad \text{Eq. (4)}$$

$$Confidence\ (X \Rightarrow Y) = \sum_k \frac{SL(X,Y)}{SL\ (X)}, \quad \text{Eq. (5)}$$

where the summation is over all *k* transactions that have an implication on the itemsets. The proposed *support* and *confidence* defined under the *SL* measure are different from those of the original association rule mining, where the calculation depends on the number of transactions that the items co-occur. In our approach, we use the $SL_{\max}$ as the divisor to calculate the percentage value of the *support* measure, whereas in the original association rule mining, the divisor is the number of transactions in the data set.

Now we give an example of generating the association rules based on the proposed *support* and *confidence* measures. Using the inter-relationship sets and the transaction records from Table 1 and Table 2, respectively, the sets of *m*-value and restricted uncertainty intervals $\hat{Un}(x)$ can be generated using the same method described in Example 1.

Using the *support* measure from Eq. (4), the itemsets with the *support* values are shown in Table 3. As a comparison, the *support* values based on the original association rule mining method are also shown in the same table. Note that the *support* values for the original association rule mining are calculated based on the number of transactions that the item(s) in the itemset co-occur without considering the multiplicity and the inter-relationships among the items. As an example, we consider the itemset, *i*(dairy,snack). Using the original association rule mining method, the support value for *i*(dairy,snack) is *63.6%*. This value is calculated by dividing the number of transactions where dairy and snack co-occurred, by the total number of transactions, i.e., (7/11) x 100%. Using our proposed approach, the *support* value can be calculated as follows.

$$Support\ (dairy \Rightarrow snack) = \sum_{k=11} SL\,(dairy, snack)$$

$$= -\frac{1}{c}\sum_{k=11} \begin{bmatrix} Bel(\{dairy, snack\}) \log_2 Bel(\{dairy, snack\}) \\ + Pl(\{dairy, snack\}) \log_2 Pl(\{dairy, snack\}) \end{bmatrix}$$

The result from the above calculation is then divided by $SL_{\max}$ whose value is *3.46* by using Eq. (3) where *n=11*, and multiplied by *100%*. This final result is equal to *78.1%*.

To generate the rules, user needs to specify the minimum *support* and *confidence* values. Suppose the *support* value is *70%*. This means by using our approach, *5* of the above itemsets pass this user-specified *support* value as shown below.

i(snack),
i(toiletry),
i(dairy),
i(dairy, snack), and
i(snack, toiletry).

We only consider the rules where both pre-condition and post-condition are non-empty, i.e., they contain at least one item. Using the original association rule mining method with the *support* value of *70%* will only generate the itemsets with singleton. Therefore, there is no rule generated under our non-empty condition assumption.

itemset	Original approach	DS-theory approach
i (dairy)	72.7%	82.1%
i (snack)	81.8%	87.7%
i (toiletry)	72.7%	84.0%
i (dairy,snack)	63.6%	78.1%
i (dairy,toiletry)	45.5%	66.5%
i (snack,toiletry)	54.5%	74.0%
i (dairy,snack,toiletry)	36.4%	28.9%

Table 3: The support values for the itemsets based on the original association rule mining method and our proposed DS-theory approach.

One advantage of our approach over the original association rule mining method is that the repeated scanning of the database to generate the itemsets is no longer necessary since both the item multiplicity and item inter-relationships are embedded in a single *bpa*.

Then the following rules can be generated from the above itemsets.

dairy ⇒ snack (*95.1%*);
snack ⇒ dairy (*89.0%*);
snack ⇒ toiletry (*84.4%*); and
toiletry ⇒ snack (*88.1%*);

where the confidence measure for each rule is given in the parentheses on the right hand side using Eq. (5). For example, the confidence measure for the rule "toiletry ⇒ snack" is *88.1%*.

Suppose that the user-specified confidence value is *90%*, then there is only one resulting rule as shown.

dairy ⇒ snack,
with *support = 78.1%* and *confidence = 95.1%*.

Note that the *support* and *confidence* measures can be set differently according to the various applications.

4. CONCLUSION

We suggested an alternative extension to the original association rule mining algorithm by providing the capability to handle the data sets with uncertainty. The evidence theory is applied to handle the uncertainty derived from the item multiplicity and item inter-relationships. We proposed a method of constructing the *bpa* in the transaction level from the *bpa* in the inter-relationship level. Unlike the original association rule mining method that requires the repeated scanning of the database to generate the itemsets and does not capture the multiplicity of an item, both the item multiplicity and item inter-relationships are embedded in a single *bpa* so that no repeated database scanning is required in the proposed framework. In addition, the *support* and *confidence* measures were defined using a Shannon-like measure of total uncertainty for the proposed framework. A numerical example of the market basket analysis (MBA) problem was given to show the result of association rule mining from data sets with uncertainty. A comparison with the original association rule mining was also given.

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