Geometric Spanner Networks: Open Problems

Giri Narasimhan

Florida International University
Miami, Florida, USA

http://www.cs.fiu.edu/~giri

Definition

Dilation or **Stretch Factor** (t(N)) of a network N is the maximum amount by which the distance between some pair of vertices in the network is increased.

$$t(N) = \max_{a,b \in N} \left\{ \frac{d_N(a,b)}{|ab|} \right\}$$

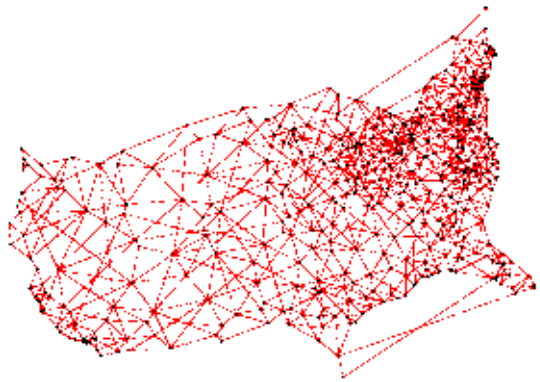
t = 10

t = 5

t = 3

t = 2

t = 1.5



t = 1.25

Spanner Problems

- Design
- Analysis
- Search
- Substructures
- Embeddings
- Miscellaneous

Spanner Problems

- Design
- Analysis
- Search
- Substructures
- Embeddings
- Miscellaneous

General Design Problem

Input: Set S of n sites, $t>1$

Output: Spanner network N connecting sites from S with stretch factor t, possibly with additional properties.

Interesting Properties:

- **Size** (# of edges), **Weight** (Total length)
- **Degree**, **Diameter**
- **Planarity**
- **Load Factor**, **Fault Tolerance**

Open: For sites in $\Re^d$, is it NP-hard to compute the sparsest (least weight) spanner?

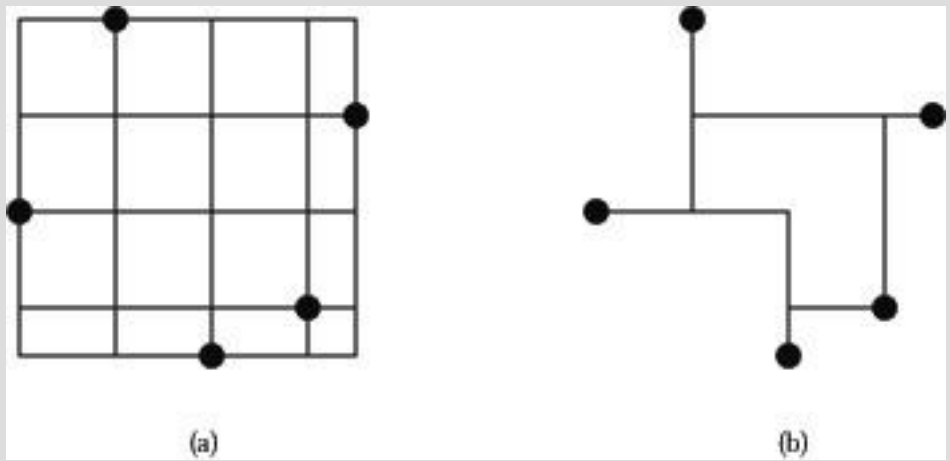
Design of t-Spanners

- **Theta graphs** O(nlogn) time, O(n) space
  [Clarkson 87, Keil 88, Althofer et al. 93]
- **Well-separated pair decomposition**
  O(nlogn) time, O(n) space [Callahan & Kosaraju 95]
- **Greedy algorithms** $O(n^2 \log n)$ time, O(n) space
  [Bern 89, Althofer et al. 93] Weight O(1) • wt(MST)
- **Greedy algorithms with Clustering**
  O(nlogn) time, O(n) space, Weight O(1) • wt(MST)
  [Gudmundsson, Levcopoulos, Narasimhan 00]

PTAS for spanners

Open: Design a PTAS for optimal weight t-spanners ?

Open: Design a PTAS for Minimum Manhattan networks?



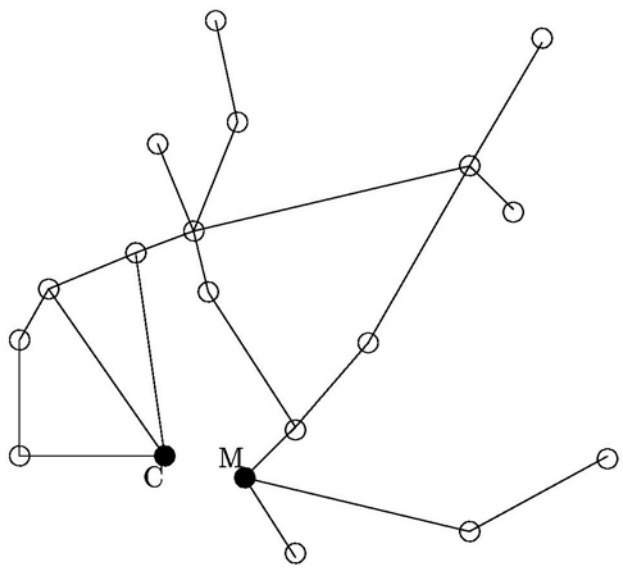
Spanner Problems

- Design
- Analysis
- Search
- Substructures
- Embeddings
- Miscellaneous

Computing Stretch Factors

Input: A geometric graph N on a set S of n sites

Output: Compute the stretch factor of N.



(1+**ε**)-approx.

O(m+nlogn) time

O(nlogn) space

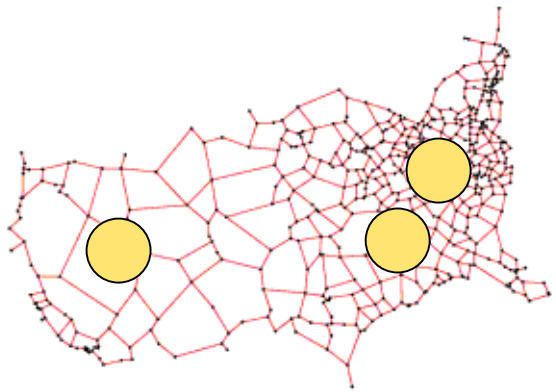
[Narasimhan & Smid '00]

Critical edges in a network?

- Open: Which edge to add to have the most impact on the stretch factor? Which k edges to add to have the most impact on SF? Which set of edges to add of total length at most B to have the most impact on SF?
- Open: The deletion of which edge can have the most increase in SF?

Fault Tolerance

- Open: How to build spanners that are tolerant to "area faults" (circular areas of fixed radius R centered anywhere in space).
- Open: What is the most "critical" area fault possible for a given network?



Geometric Analysis

Input: Set S of n sites; Set E of edges joining sites;
Property P Satisfied by E

Output: wt(E) $\leq$??

- Theta Graph Property [Clarkson, Keil]
- Diamond Property [Das]
- Gap Property [Das, Narasimhan]
- Leapfrog Property [Das, Narasimhan]
- Isolation Property [Das, Narasimhan]

k-OPT algorithm for TSP

- 2-OPT has worst-case performance ratio of $\Omega(\lg n / \lg \lg n)$
- k-OPT has worst-case performance ratio of $O(\lg n)$ [Using the Gap Property]
- Open: How to analyze k-OPT?
- Open: Identify new properties

Spanner Problems

- Design
- Analysis
- Search
- Substructures
- Embeddings
- Miscellaneous

Shortest Path Queries in Spanners

- Let N be a geometric t-spanner for a set S of n points in $\Re^d$ with m edges. N can be preprocessed so that $(1+\varepsilon)$-approximate shortest path lengths between two query points from S can be reported efficiently.
  - Preprocessing O(m + nlogn)
  - Space O(m + nlogn)
  - Query O(1)

Caveat: Interpoint distances need to be in range: $[Dn^{-k}, D]$

[OPEN] Eliminate this condition?

Spanner Problems

- Design
- Analysis
- Search
- Substructures
- Embeddings
- Miscellaneous

Optimal substructures in spanners

- Open: Given a set of n sites, design an algorithm to compute a
  - tree with the least (or approx.) stretch factor? [Eppstein]
  - planar graph with the least (or approx.) stretch factor?
  - path with the least (or approx.) stretch factor?

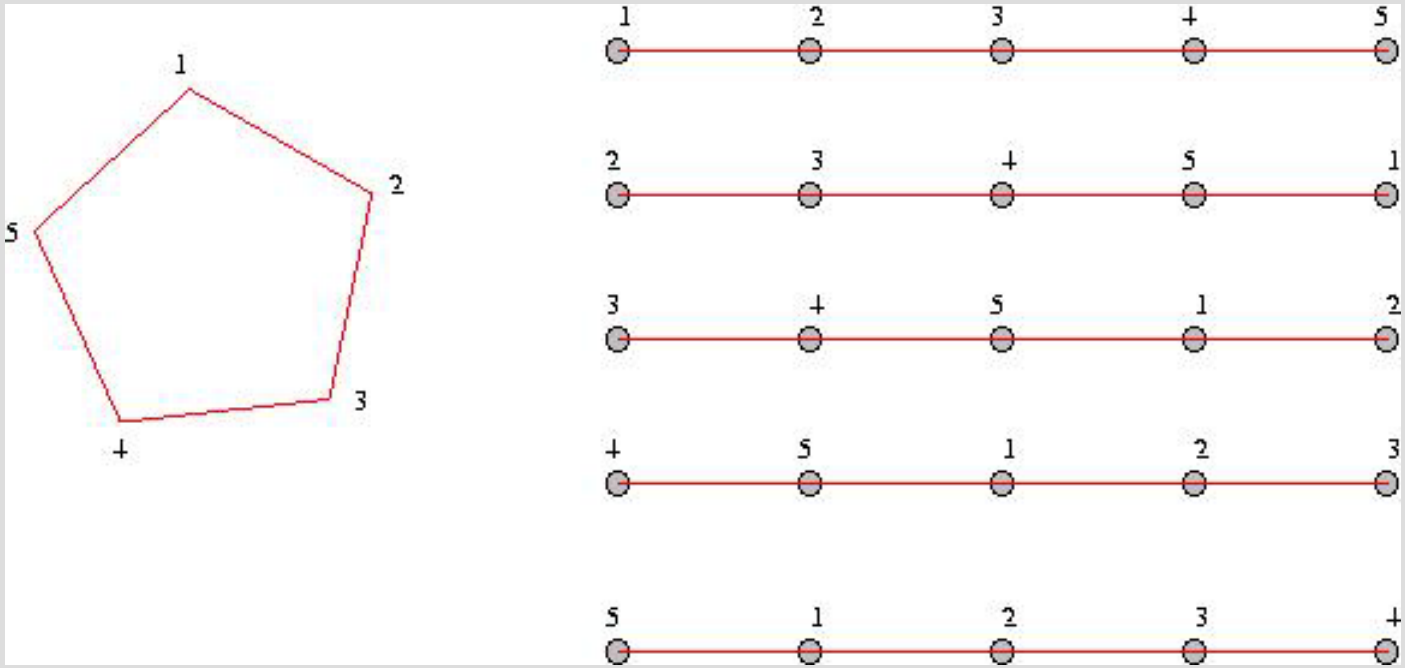
Spanner Problems

- Design
- Analysis
- Search
- Substructures
- Embeddings
- Miscellaneous

Probabilistic Embeddings

- How to embed a n-path in Euclidean space?
  - Easy! Embed on a line.
- How to embed a n-cycle in Euclidean space with small distortion in distances?
- There is a good probabilistic embedding! [Karp]
  - Approximate it by n path metrics. Distortion = 2

Embedding a cycle



Probabilistic Embeddings

- Probabilistic Embeddings [Bartal]
- Open: Find good probabilistic embeddings of tree metrics in Euclidean space.

Embedding Results

	Euclidean	Tree Distributions
General	$\Theta(\lg n)$	$O(\lg n \lg\lg n)$
Planar	$O((\lg n)^{1/2})$ [Rao99]	$O(\lg n)$
Series-Parallel	$O(1)$ [Gupta+99]	$\Omega(\lg n)$
Grid		$\Omega(\lg n)$ [Alon+95]
Tree	$\Omega((\lg \lg n)^{1/2})$	

Spanner Problems

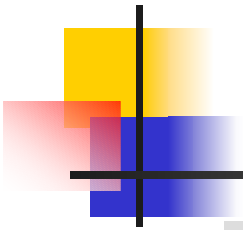
- Design
- Analysis
- Search
- Substructures
- Embeddings
- Miscellaneous

Miscellaneous

- Open: How to efficiently construct spanners for point sets in the presence of obstacles?

Random Thoughts

- New Network Models?
  - Wireless Networks? Optical Fiber Networks?
- Heterogeneous Networks
  - Finite # of Types of Links? Types of Nodes?
- Other Cost Metrics?
  - Combination of metrics?



Thanks!

New book

Geometric Spanner Networks

by

Giri Narasimhan & Michiel Smid