COT 5407 Introduction to Algorithms

Homework 3

DUE: Tuesday, Oct 2, 2012 11AM

Please remember that all submissions are to be your own work and must be typeset. Include your name. Handwritten submissions will NOT be accepted.

1. Textbook problem 8-4, page 206 (Water jugs).

2. There is a prize hidden in a box; the value of the prize is a positive integer between 1 and N, and you are given N. To win the prize, you have to guess its value. Your goal is to do it in as few guesses as possible. You start with a number of chips (specified below). Each chip allows you one guess that's too high. If you guess too high, and you have no chips, you lose. So, for example, if you start with no chips, then you can win in N guesses by simply guessing the sequence 1, 2, 3, ...

   Suppose you start with 1 chip. Describe a strategy that makes $O(\sqrt{N})$ guesses.

3. Use the Carter-Wegman trick to determine the value of 1463528785364712 mod 99999999 without using any calculating devices or long division. You have a limit of ten lines for your answer.

4. Suppose you have N rational numbers $r_1, r_2, r_3, ...r_N$, whose numerators and denominators are all between 1 and N; you may assume all the rational numbers have value at least 1. Show how to sort these numbers in linear time, assuming that any operations on the numerators or denominators (adds, subtracts, multiplies, integer divides, and integer mod operations) take constant time. *Hint: Linear-time sorts are generally done by radix sort. Suppose $r_i - r_j > 0$. What is the minimum value of $r_i - r_j$?*