COT 5407 Introduction to Algorithms

Homework 5

DUE: Thursday, November 29, 2012

I would like to strongly reiterate that all submissions are to be submitted as typeset PDF files.

1. A double-ended queue is a data structure that supports insertion and deletion at both ends of the queue. Suppose we add the `findMin` operation to the double-ended queue; `findMin` returns (but does not remove) the smallest item in the double-ended queue. Design a data structure that implements the double-ended queue in $O(1)$ amortized cost for all operations. You must prove your time bound.

2. Let A be an N-by-N matrix of zeros and ones. A submatrix S of A is any group of *contiguous* entries that form a square. Using dynamic programming, design an $O(N^2)$ algorithm that determines the size of the largest submatrix of ones in A. For instance, in the matrix that follows, the largest submatrix is a 4-by-4 square.

```
10111000
00101000
00111000
00111010
00111111
01011110
01011111
00011110
```

3. Given an *undirected* graph $G = (V, E)$, and an integer K, the **ARC-DELETION** problem is that of determining if there is a set of K edges whose deletion breaks all cycles. Either give a polynomial-time algorithm or prove that ARC-DELETION is NP-complete.

4. A coin collector has an opportunity to purchase N coin collections $S_1, S_2, ..., S_N$. Some collections may contain coins in common with others. The collector wants to buy at least K collections out of the N, but **must avoid duplicates**. The **COIN COLLECTOR'S PROBLEM** is whether there is a selection of at least K mutually disjoint collections out of the N. Either give a polynomial-time algorithm or prove that COIN COLLECTOR'S PROBLEM is NP-complete.