Data Structures

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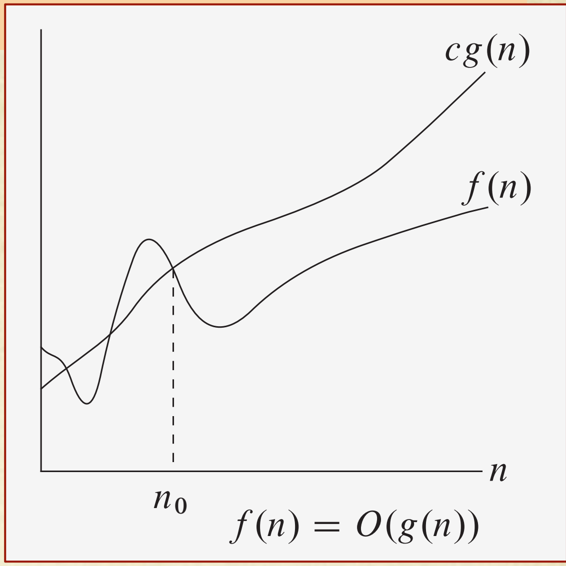
Review

How to compare big-Ohs?



- We say that f(n) = O(g(n)) if
  - There exists positive integer n_0, and
  - A positive constant c, such that
  - For all $n \geq n_0$,

    $f(n) <= c\ g(n)$
- In other words, for all large enough values of n, there exists a constant c, such that
  - f(n) is always bounded from above by cg(n)
- Note that the condition may be violated for a finitely many small values of n

Summary

- To prove $f(n) = O(g(n))$
  1. Pick a value of c
  2. Find a n_0 such that
  3. $f(n) \leq c\, g(n)$ for all $n \geq n_0$
  4. If not, go back to step 1 and refine value of c

- To prove $f(n) \neq O(g(n))$
  - Assume that c is fixed to some positive value
  - And assume there exists a n_0 such that
  - $f(n) \leq c\, g(n)$ for all $n \geq n_0$
  - Now try to prove a contradiction to this claim

Challenging Cases

MAXSUBSEQSUM(A)

Initialize maxSum to 0

N := size(A)

For i = 1 to N do

    For j = i to N do

        Initialize thisSum to 0

        for k = i to j do

            add A[k] to thisSum

        if (thisSum > maxSum) then

            update maxSum

$$\sum_{k=i}^{j} 1 = j - i + 1$$

$$\sum_{j=i}^{N} (j - i + 1) = \frac{(N-i+1)(N-i+2)}{2}$$

$$\sum_{i=1}^{N} \frac{(N-i+1)(N-i+2)}{2}$$

$$= \sum_{i=1}^{N} \frac{i^2}{2} - (N + \frac{3}{2}) \sum_{i=1}^{N} i + \frac{1}{2}(N^2 + 3N + 2) \sum_{i=1}^{N} 1$$

$$= \frac{N^3 + 3N^2 + 2N}{6} = O(N^3)$$

Review

Example 1

```
SampleMethod1 (N)
For i = 0 to N do
    For j = i to N do
        sum = i + j
Report sum
```

Big Oh: _______________

Example 2

```
public void sampleMethod2(int[] a)
{
    intindex = 0;
    for(int i = a.length-1; i >= 0; i--)
        if(a[i] > target && a.length > 0)
            index = i;
}
```

Big Oh: _______________

Review

Example 3

```
public void sampleMethod3() {
    System.out.println("This is sample 3.");

    int b = 10;
    int c = 15;
    int d = b – c;
    b = b * c;
    d = d * b;
    c = d + d;
    b = c + d + b + d;
    System.out.println(b + " " + c + " " + d);
}
```

Big Oh: ____________

Example 4

```
public void sampleMethod4(int n) {

for(int i = 0; i <= n; i++)
    for(int j = 1; j <= n; j = j * 2)
        for(k = 0; k <= n/2; k++)
            System.out.println(i + j + k);
}
```

Big Oh: ____________

Linear Data Structures

Linear Data Structures

- Lists
  - ArrayList
  - LinkedList
- Stacks
- Queues

Applications of Stacks

- Balancing Parentheses
  - Is this balanced?
    - (()((()()())()))
  - Push when you see an open parenthesis.
  - Pop when you see a closed one.
  - If stack empties at end of string, it is balanced.
  - Else, it is not.
- Postfix Expressions
  - How to evaluate this:
    - 4 2 * 5 + 7 + 1.5 *
  - Push when you see a number
  - Pop two when you see an operator, operate and push result

How to deal with (unary) negative signs such as 4 * (-5)?

Applications of Stacks … 2

- Infix to Postfix Conversion
  - How to convert this:
    - **Infix**: ((4 * 2) + 5 + 7) * 1.5 **to Postfix**: 4 2 * 5 + 7 + 1.5 *
  - **Infix**: 4 * 2
  - Idea: Push operators on a stack and print operands
  - **Infix**: 4 + 2 + 3
  - Idea: Before pushing operators, pop previous operator
  - **Infix**: 4 + 2 * 5 + 3
  - Idea: Before pushing operators, pop all operators of higher or equal precedence
  - **Infix**: 4 * (2 + 3) + 5
  - Idea: Push open parenthesis on stack. When closed parenthesis is encountered, pop stack until open parenthesis is popped

Applications of Queues

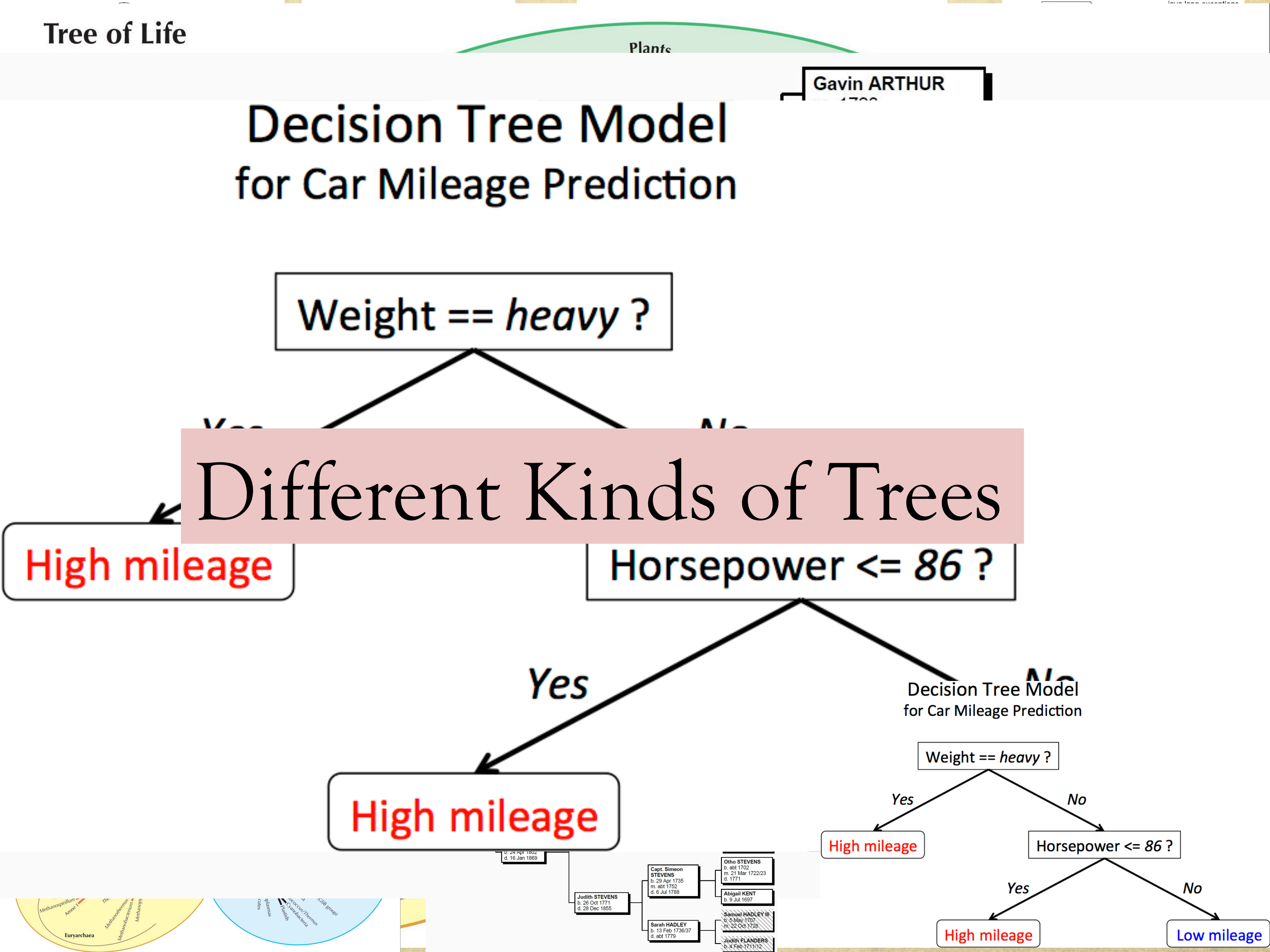
- Servers
  - Disk server, File Server, Print Server, …

Hierarchical Data Structures

Trees were invented to …

- Store hierarchical information
  - File system
  - Software Hierarchy
  - Administrative Hierarchy
  - Geographical Hierarchy
  - Decision trees
  - Parse trees
  - Family genealogy
  - Tree of life
  - …
- To deal with **inefficiencies of Linear Data Structures** and to store **dynamic information**
  - To be explained later!

Different Kinds of Trees



Tree

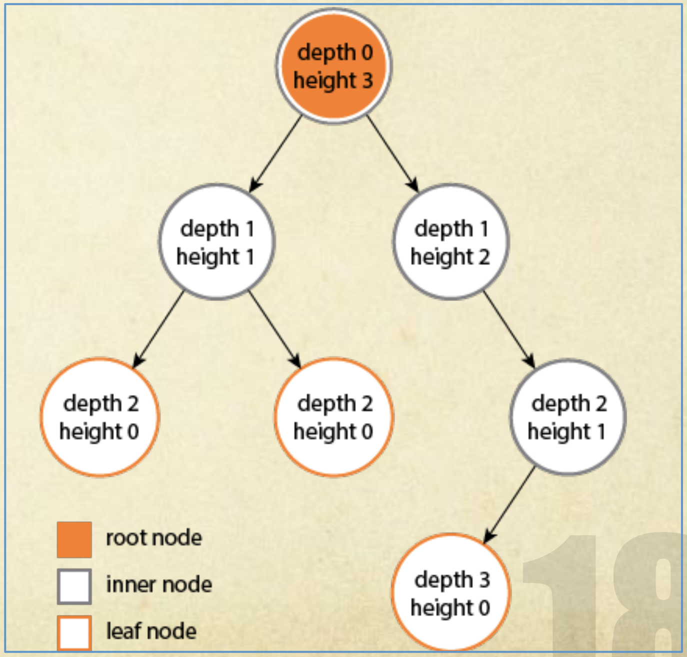
- **Def**: Connected hierarchical structure without cycles
- **Def**: A tree is an abstract mathematical object with
  - Set of **nodes**, V, with special **root** node, r
  - Set of **directed edges**, E, such that for each edge e in E, e = (u, v), where u is **parent** of v, or v is a child of u, and
  - Every node has a **unique parent** except the root node, r.
- **Def**: A **binary** tree is a tree where every node has at most two children
- **Def**: A **subtree** of a tree, T = (V,E) is the tree rooted at some node from V.

Tree Terminology

- **Root**: node with no parent
- **Leaf and internal node**: node with no child and with at least one
- **Parent**, **Child**: each directed edge defines parent-child
- **Sibling**: a node that shares the same parent
- **Ancestor (Descendant)**: all nodes on path to root (leaf)
- **Left child**, **right child**: if children are ordered, then this terminology is relevant in binary trees
- **Full binary tree**: each node has 2 children or is a leaf
- **Complete binary tree**: tree filled level by level, left to right

Tree Terminology

- **Depth of node**: length of path from root to node
- **Height of tree (or subtree)**: length of longest path from root to leaf
- Like a linked list, a tree can be defined recursively
  - Subtrees rooted at nodes are also trees



Properties of Trees

- If n is the number of nodes in a tree, then
  - Number of edges in a tree = n-1
- Number of leaves in a full binary tree = 1 more than number of internal nodes
- Number of empty subtrees = n+1

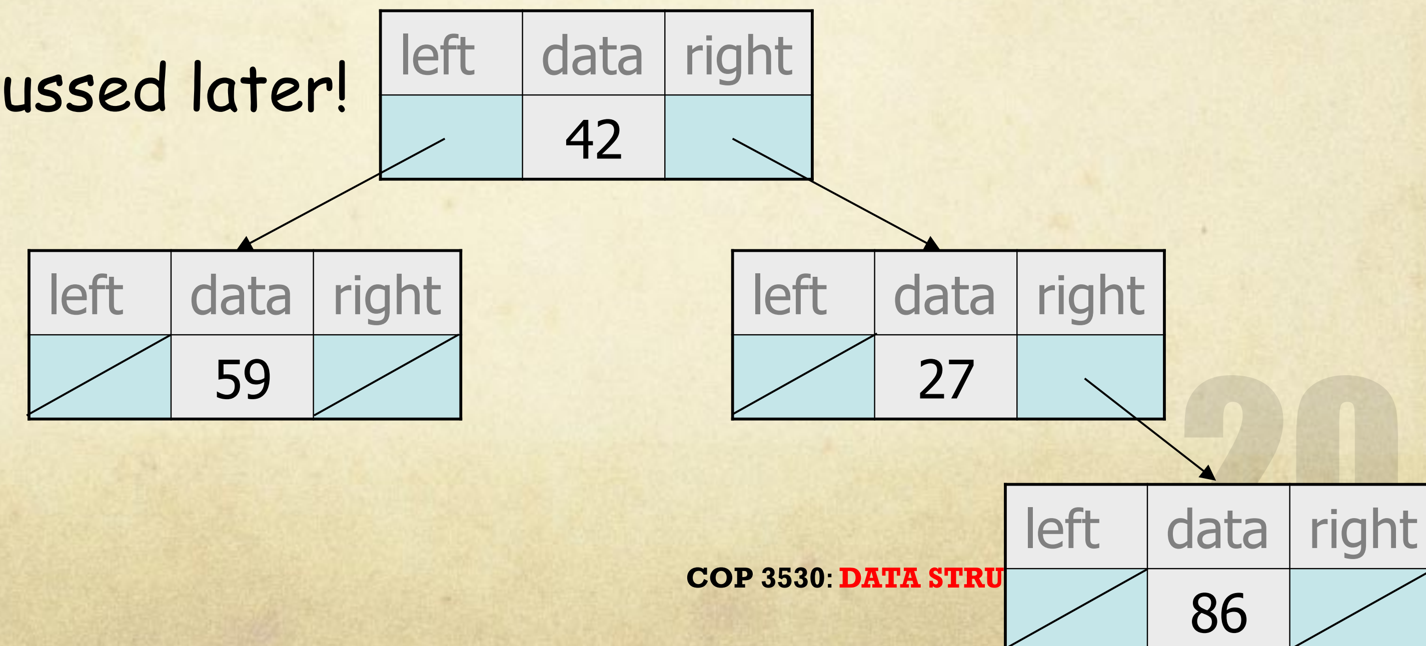
Implementing Binary Trees

Generalizing linked lists

- Instead of next/prev pointers, each node has 2 pointers
  - Left and right child; if needed, pointer to parent

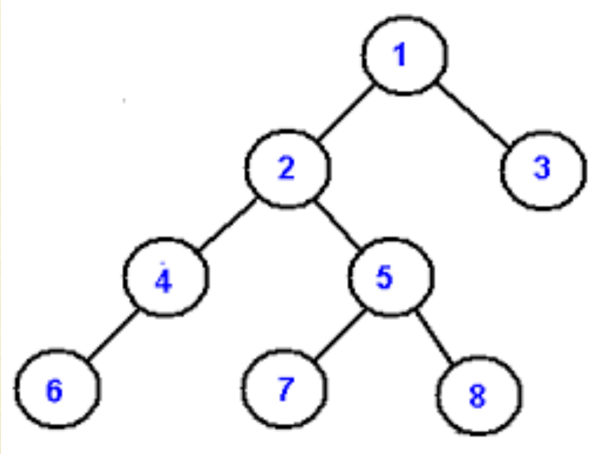
Can be implemented using Arrays

- To be discussed later!



Tree Traversals

- Inorder traversal
  - Left subtree – Node – Right subtree
  - 6, 4, 2, 7, 5, 8, 1, 3
- Preorder traversal
  - Node – Left subtree – Right subtree
  - 1, 2, 4, 6, 5, 7, 8, 3
- Postorder traversal
  - Left subtree – Right subtree – Node
  - 6, 4, 7, 8, 5, 2, 3, 1



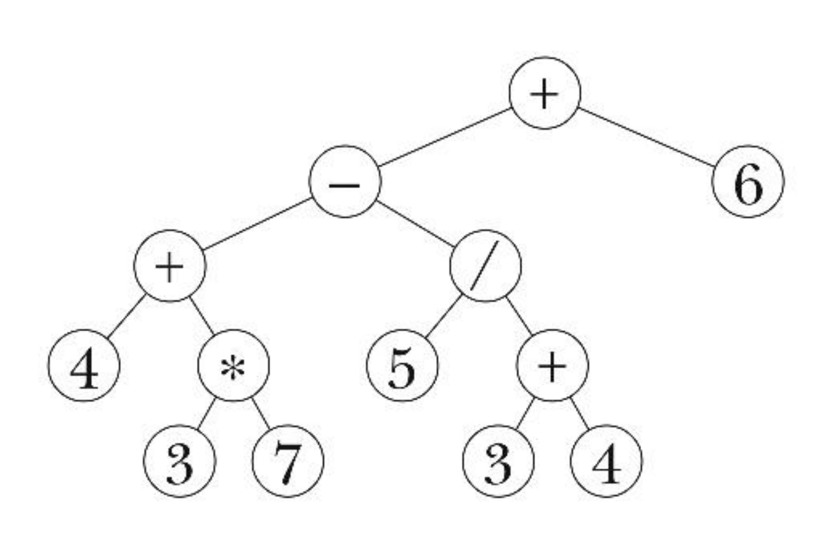
https://www.cs.cmu.edu/~adamchik/15-121/lectures/Trees/pix/non_rec_trav.bmp

Implementation

```
static void preorder(BinaryNode rt) {
  if (rt == null) return; // Empty subtree - do nothing
  visit(rt);                       // Process root of subtree
  preorder(rt.left());    // Process all nodes in left subtree
  preorder(rt.right());   // Process all nodes in right subtree
}
```

Tree Traversal Applications

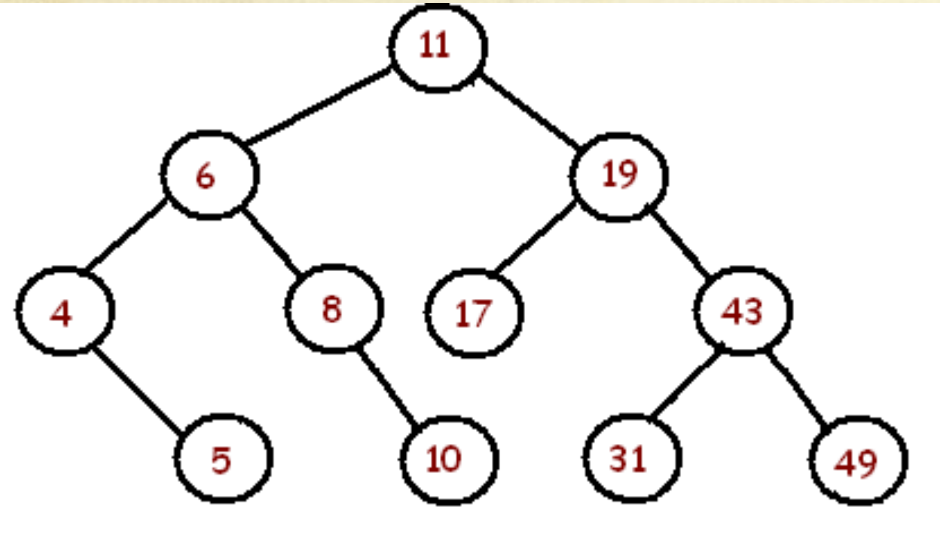
- Expression Parse trees
  - Inorder traversal?
  - Postorder traversal?
- Printing directory structure
  - Figure 4.7 in Weiss book



https://people.eecs.berkeley.edu/~bh/ss-pics/parse0.jpg

Binary Search Trees

- Binary Tree where each node stores a value
- Value stored at node is larger than all values stored in nodes of left subtree
- Value stored at node is smaller than all values stored in nodes of right subtree



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