COT 5407: Introduction to Algorithms

Giri NARASIMHAN

www.cs.fiu.edu/~giri/teach/5407S19.html

Momentos

- Slides and Audio online
- Need to register
  - Go to https://fiu.momentos.life
  - If you don't already have an account
    - Click on "Sign up"
    - Follow instructions & use referral code: XLY6FD
  - If you have an account, "Add Course" with course name and referral code XLY6FD
  - Verify account using link sent to email

Person of the Year …

The first hundred votes …

Who won a majority?

48	12	9	12	23	12	22	12	12	12
48	93	93	93	12	12	93	12	93	12
12	93	48	48	12	12	12	33	79	12
12	12	93	12	12	9	12	23	12	12
12	12	12	33	93	93	93	12	12	12
12	9	12	23	93	48	48	12	12	44
93	93	93	12	12	9	12	23	12	55
12	12	48	12	48	48	12	48	88	12
12	12	93	12	12	9	12	23	12	12
12	12	12	33	93	93	93	12	12	12

Every number in the table corresponds to a vote for a person with that ID

Majority: More than 50% of the votes

Standard Approaches

- Keep a list of candidates and their counts
  - Every vote needs to be compared against every candidate in the worst case
- Sort the list and count
  - Sorting is the bottleneck
  - Can we avoid sorting?

Wacky Ideas, anyone?

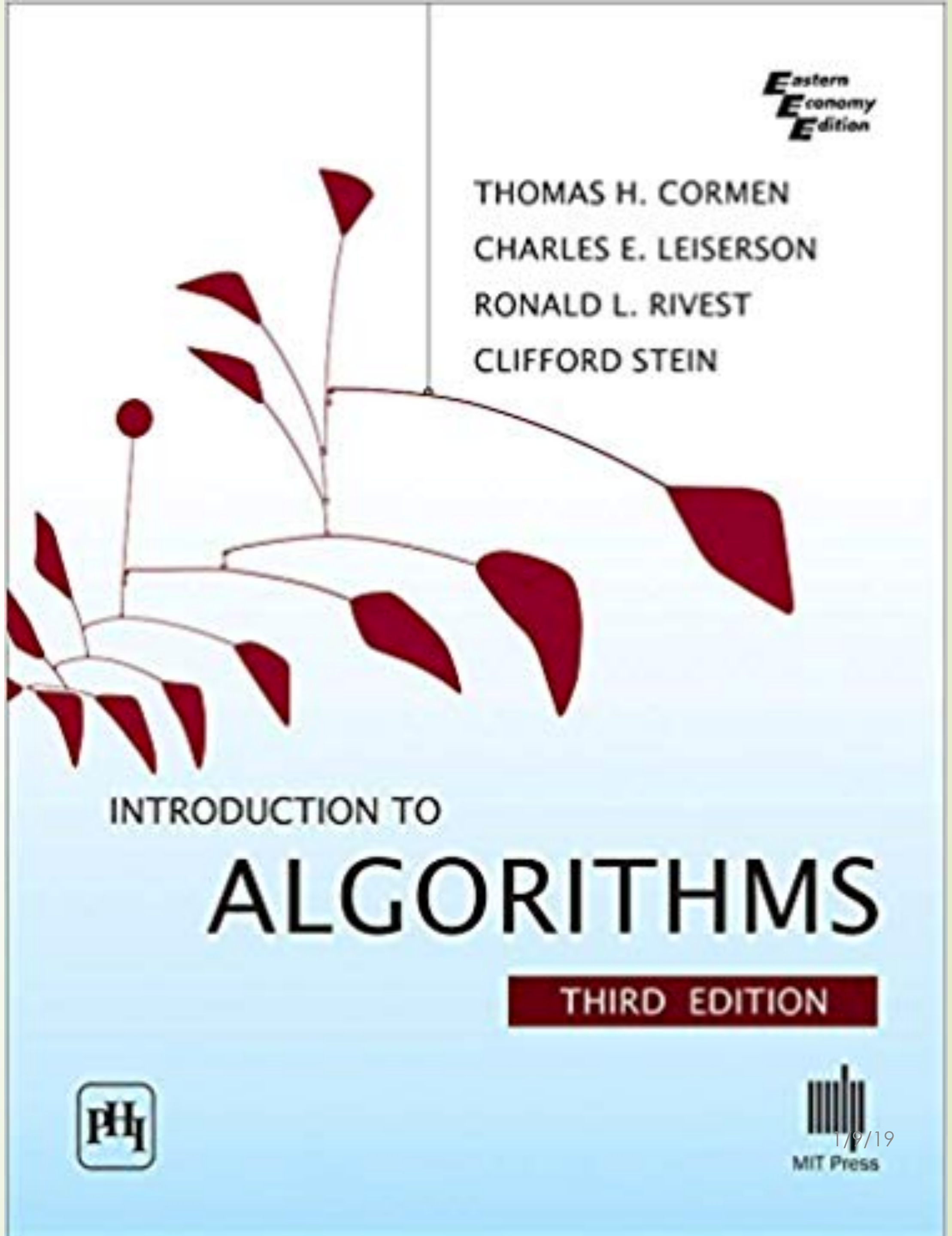
- What if I pick two random votes and they turn out to be different?
  - Discard and reduce the problem size
- What if I pick two random votes and they are the same?
  - Well, this needs work and you will need to think about it!

48	12	9	12	23	12	22	12	12	12
48	93	93	93	12	12	93	12	93	12
12	93	48	48	12	12	12	33	79	12
12	12	93	12	12	9	12	23	12	12
12	12	12	33	93	93	93	12	12	12
12	9	12	23	93	48	48	12	12	44
93	93	93	12	12	9	12	23	12	55
12	12	48	12	48	48	12	48	88	12
12	12	93	12	12	9	12	23	12	12
12	12	12	33	93	93	93	12	12	12

Text

3rd Edition

- ISBN-13: 978-8120340077
- ISBN-10: 9788120340077



Evaluation

Exams (2)	50%
Quizzes	10%
HW Assignments	30%
Semester Project	5%
Class Participation	5%
Kattis Submissions	5% (Extra Credit)

Kattis

- Repository of problems
- Programming solutions can be uploaded
- Build a profile of problems solved by you
- Weekly mock competitions on Saturdays
  - E.g., FIU-SCIS-12JAN2019 (from noon to 5 PM)

What you should already know …

- Array Lists
- Linked Lists
- Sorted Lists
- Stacks and Queues
- Basic Sorting Algorithms
- Trees
- Binary Search Trees
- Heaps and Priority Queues
- Graphs
  - Adjacency Lists
  - Adjacency Matrices

History of Algorithms

- **Euclid**, 300 BC
- **Bhaskara**, 6th c
- **Al Khwarizmi**, 9th c
- **Fibonacci**, 13th c
- **Gauss**, 18-19th c
- **Babbage**, 19th c
- **Turing**, 20th c
- **von Neumann**, 20th c
- **Knuth**, **Karp**, **Tarjan**, **Rabin**, …, 20-21st c

Gauss – sum of series

- 1 + 2 + 3 + … + N
- Gauss observed that
  - 1 + N = N+1
  - 2 + N-1 = N+1
  - …
- Thus,
  - 1 + 2 + 3 + … + N
  - = (2 + 3 + … + N-1) + (N+1)
  - = (3 + … + N-2) + (N+1) + (N+1)

- Keep reducing until when?
  - Depends on whether N is even/odd
  - If N is even:
    - = (N+1) N/2 = N(N+1)/2
  - If N is odd:
    - = (N+1) (N-1)/2 + (N+1)/2 = N(N+1)/2

Al Khwarizmi's algorithm

- 43 X 17
  - 43 17
  - 21 34
  - 10 68 (ignore)
  - 5 136
  - 2 272 (ignore)
  - 1 544

731

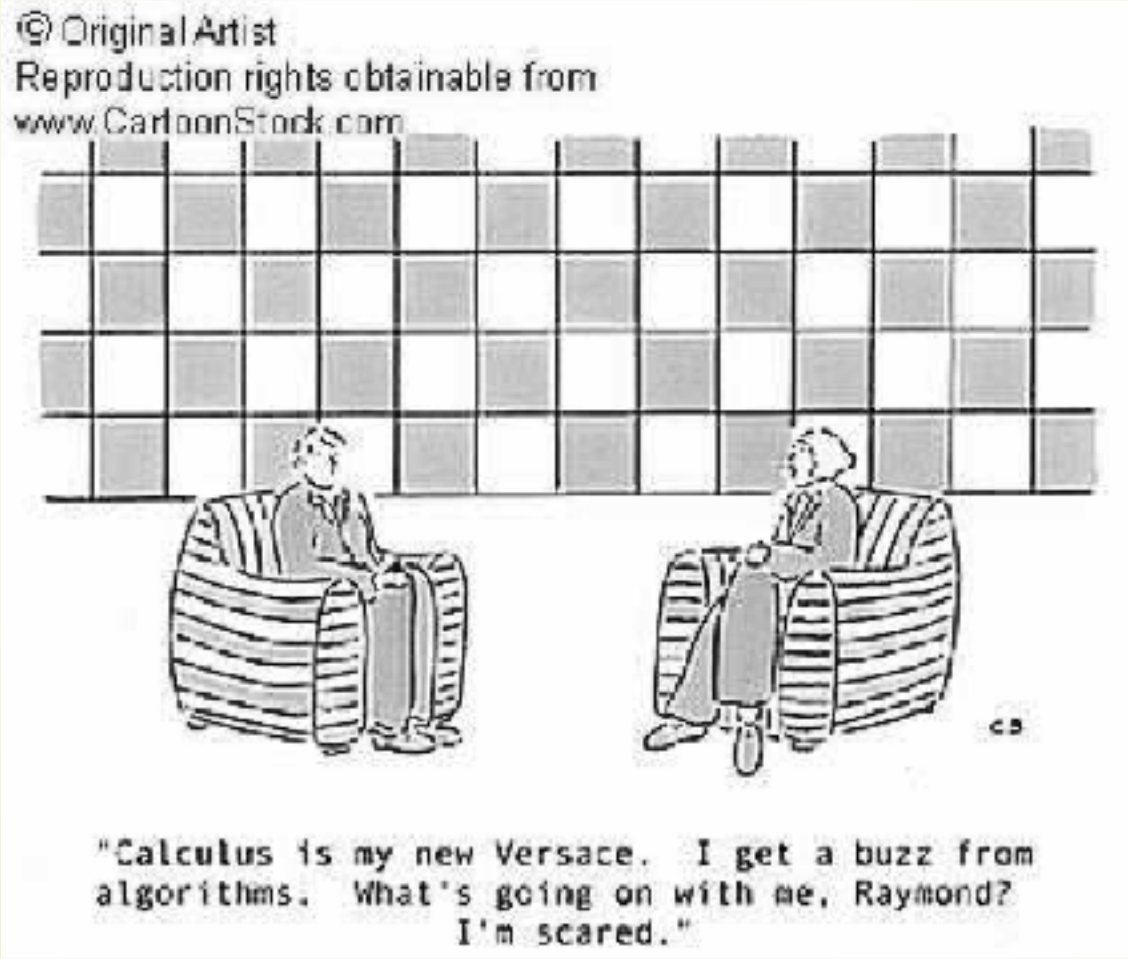
Euclid's Algorithm

- GCD(12,8) = 4; GCD(49,35) = 7;
- GCD(210,588) = ??
- GCD(a,b) = ??
- **Observation**: [a and b are integers and a ≥ b]
  - GCD(a,b) = GCD(a-b,b)
- **Euclid's Rule**: [a and b are integers and a ≥ b]
  - GCD(a,b) = GCD(a mod b, b)
- **Euclid's GCD Algorithm:**
  - GCD(a,b)

If (b = 0) then return a;

return GCD(a mod b, b)

If you like Algorithms, nothing to worry about!



Search

- You are asked to guess a number X that is known to be an integer lying in the range A through B. How many guesses do you need in the worst case?
  - Use binary search; Number of guesses = $\log_2(B-A)$
- You are asked to guess a positive integer X. How many guesses do you need in the worst case?
  - NOTE: No upper bound is known for the number.
  - Algorithm:
    - figure out B (by using Doubling Search)
    - perform binary search in the range B/2 through B.
  - Number of guesses = $\log_2 B + \log_2(B – B/2)$
  - Since X is between B/2 and B, we have: $\log_2(B/2) < \log_2 X$,
  - Number of guesses < $2\log_2 X - 1$

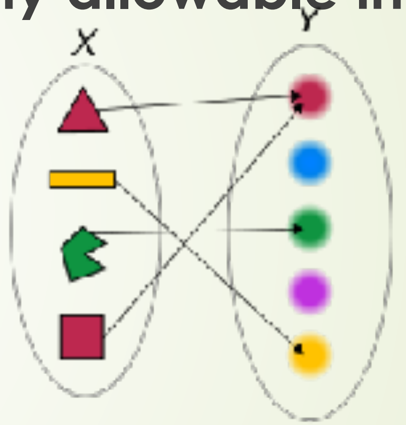
Polynomial Evaluation

- Given a polynomial
  - $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_{n-1} x^{n-1} + a_n x^n$

  compute the value of the polynomial for a given value of x.
- How many additions and multiplications are needed?
  - Simple solution:
    - Number of additions = n
    - Number of multiplications = 1 + 2 + … + n = n(n+1)/2
  - Reusing previous computations: n additions and 2n multiplications!
  - Improved solution using Horner's rule:
    - $p(x) = a_0 + x(a_1 + x(a_2 + \ldots x(a_{n-1} + x\, a_n))\ldots))$
    - Number of additions = n
    - Number of multiplications = n

Definitions

Abstract Problem: defines a function from any allowable input to a corresponding output



Instance of a Problem: a specific input to abstract problem

Algorithm: well-defined computational procedure that takes an instance of a problem as input and produces the correct output

An Algorithm must halt on every input with correct output.

Sorting

- Input is a sequence of n items that can be compared.
- Output is an ordered list of those n items
  - I.e., a reordering or permutation of the input items such that the items are in sorted order
- Fundamental problem that has received a lot of attention over the years.
- Used in many applications.
- Scores of different algorithms exist.
- Task: To compare algorithms
  - On what bases?
    - Time
    - Space
    - Other

Sorting Algorithms

- Number of Comparisons
- Number of Data Movements
- Additional Space Requirements

Sorting Algorithms

- SelectionSort
- InsertionSort
- BubbleSort
- ShakerSort
- MergeSort
- HeapSort
- QuickSort
- Bucket & Radix Sort
- Counting Sort