SPRING 2019: **COT 5407** Intro. to Algorithms

[Homework 3; Due midnight of Feb 25 via Email or Canvas]

General submission guidelines and policies: Add the following signed statement. Without this statement, your homework will not be graded.

> I have adhered to the collaboration policy for this class. In other words, everything written down in this submission is my own work. For problems where I received any help, I have cited the source, and/or named the collaborator.

Note that all solutions to dynamic programming (DP) problems must show (a) the hierarchy of subproblems required to solve the problem, (b) the recurrence relation connecting the solutions of these subproblems along with some explanation for it, (c) the description of the data structure that will store solutions to previously solved subproblems, (d) the actual algorithm (i.e., detailed pseudocode), and (e) the time complexity analysis. State clearly any reasonable assumptions you make (as long as your assumptions don't oversimplify the problem).

Problems

18. (**Exercise**) Wite down the invariant for Partition from page 171 from [CLRS].

19. (**Regular**) Given an array B with n items from the range $[1, n]$, and given that it is the output you get from calling Randomized-Partition$(A, 0, n-1)$ (see page 179 from [CLRS]), design a $o(n^2)$ (sub-quadtratic) algorithm called FindAllPivots$(B, 0, n-1)$ that efficiently outputs all items in B that may have been used as the pivot to produce B as the output. Note that A is not given to you.

20. (**Exercise**) Verify that if array is $B = [2, 1, 3, 6, 8, 9, 5, 7, 4]$, then FindAllPivots outputs only one item, i.e., $\{3\}$. If $B = [1, 2, 3, 4, 5, 6, 7, 9, 8]$, then it outputs 7 items, i.e., $\{1, 2, 3, 4, 5, 6, 7\}$. And if $B = [9, 8, 7, 6, 4, 5, 3, 2, 1]$, then the output is empty.

21. (**Exercise**) Argue that your algorithm FindAllPivots is correct.

22. (**Regular**) Suppose you are given two sets A and B, each containing n integers. You can choose to reorder each set however you like. After reordering, let $A[i]$ be the i-th element of set A, and let $B[i]$ be the i-th element of set B. You will then be charged an amount equal to $\sum_{i=1}^{n} A[i] * B[i]$. Give an algorithm that will minimize your charges. Prove that your algorithm minimizes the charges, and state its running time.

23. (**Exercise**) Solve 16.3-3, page 436.

24. (**Exercise**) Study Section 15.2 from the book and solve 15.1-5, page 370.

25. (**Regular**) It is the hurricane season. You have just bought a wooden board of length L (and standard width), which needs to be cut into n smaller pieces so that you can put it up as a hurricane shutter. The cuts are required to be at at locations $l_1, l_2, \ldots, l_n$ ft from the left end of the board. However, the store charges money for cutting. Their cutting rates are strange; if you cut a board of length x into two smaller pieces (of any lengths), you will be charged $\$x$.

    The cutting order will determine the cost of the cuttings required. For example, assume that your board is of length 10 ft and that you need to cut it at locations 2ft, 4ft and 7ft from the left end. If you cut it in that order, then your cost will be $\$10 + 8 + 6 = \$24$. On the other hand, if you cut it at 4ft first and then at 2ft and 7ft for the two smaller pieces, then the total cost will be only $\$10 + 4 + 6 = \$20$.

    Design an algorithm to determine the optimal order of cuts required to minimize your total cutting costs. Analyze your algorithm.