Introduction to Data Science

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Time Series

Time Series Analysis Applications

- Economic Forecasting
- Sales Forecasting
- Budgetary Analysis
- Stock Market Analysis
- Yield Projections
- Process and Quality Control
- Inventory Studies
- Workload Projections
- Utility Studies
- Census Analysis

https://www.itl.nist.gov/div898/handbook/pmc/section4/pmc4.htm

Average as an Estimate

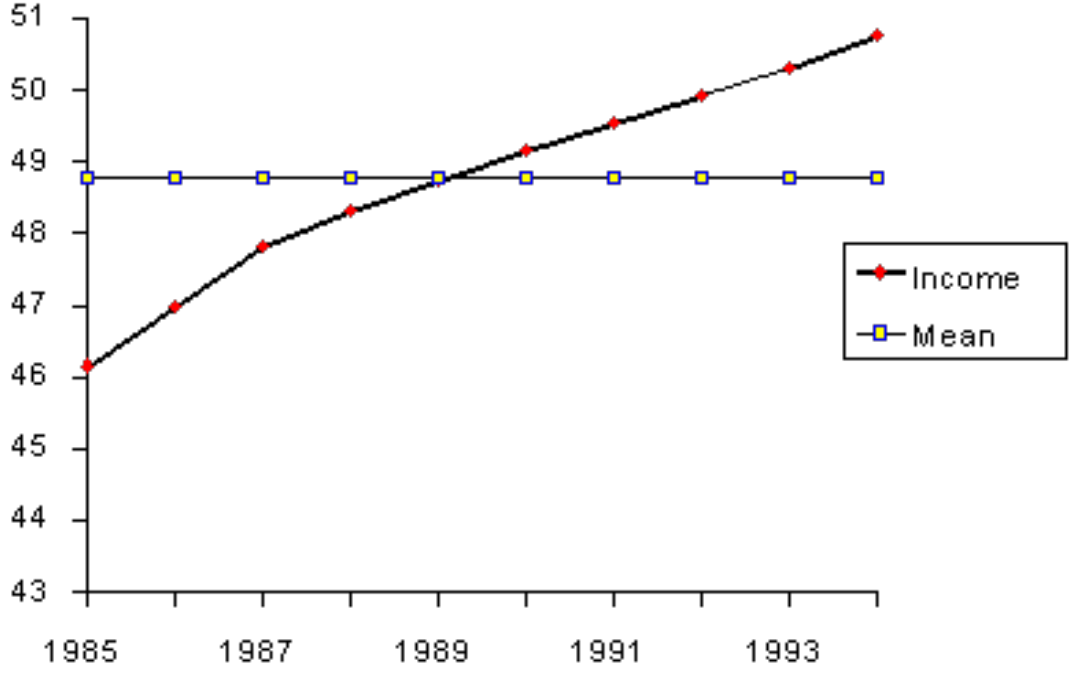
Suppl	$	Error	ES
1	9	-1	1
2	8	-2	4
3	9	-1	1
4	12	2	4
5	9	-1	1
6	12	2	4
7	11	1	1
8	7	-3	9
9	13	3	9
10	9	-1	1
11	11	1	1
12	10	0	0

Avg Est	7	9	10	12
SSE	144	48	36	84
MSE	12	4	3	7

- Is Average a good estimate?
  - Yes, it minimizes Mean Square Error (MSE)

Is Average a good predictor?

- Simple average is a bad predictor
  - Example on right: average does not show trend nor predict future



Moving Average vs Average

Suppl	$	MA	Error	ES
1	9			
2	8			
3	9	8.667	0.333	0.111
4	12	9.667	2.333	5.444
5	9	10.000	-1.000	1.000
6	12	11.000	1.000	1.000
7	11	10.667	0.333	0.111
8	7	10.000	-3.000	9.000
9	13	10.333	2.667	7.111
10	9	9.667	-0.667	0.444
11	11	11.000	0	0
12	10	10.000	0	0

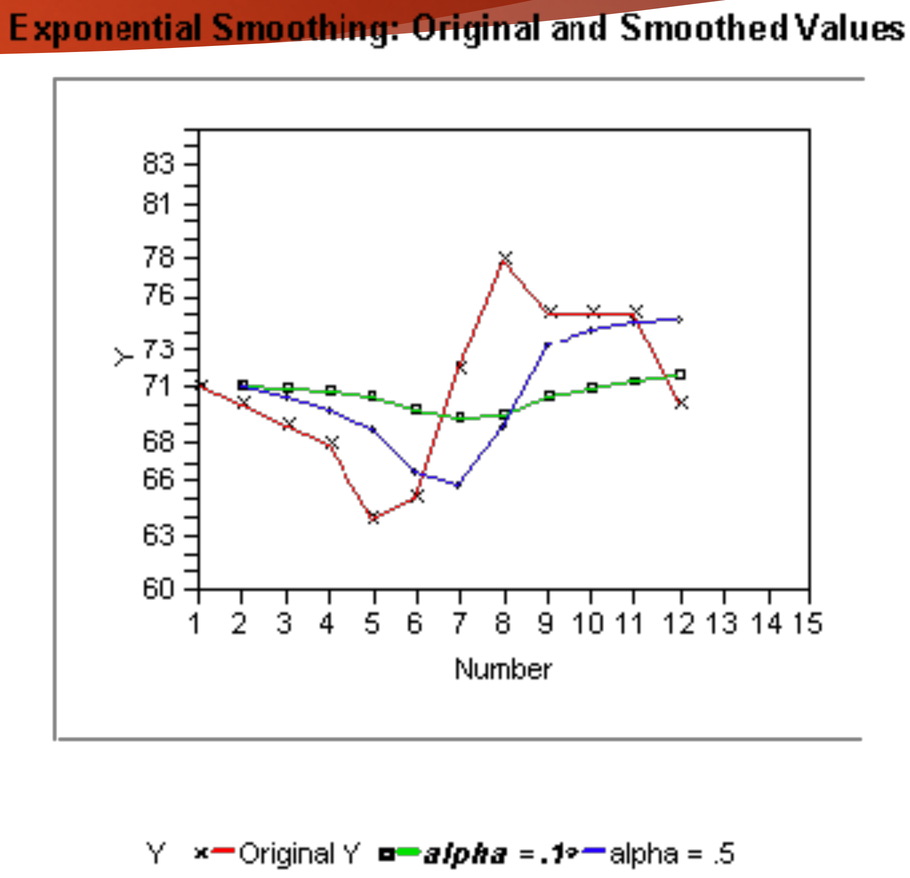
- Window size = 3
- MSE = 3.0
- Moving MSE = 2.42
- Moving Average (MA) has lower error than Average

Double Moving Averages

- Compute moving average of moving average using same window size
- Errors can be reduced further
- Do a linear regression with single and double MA to forecast

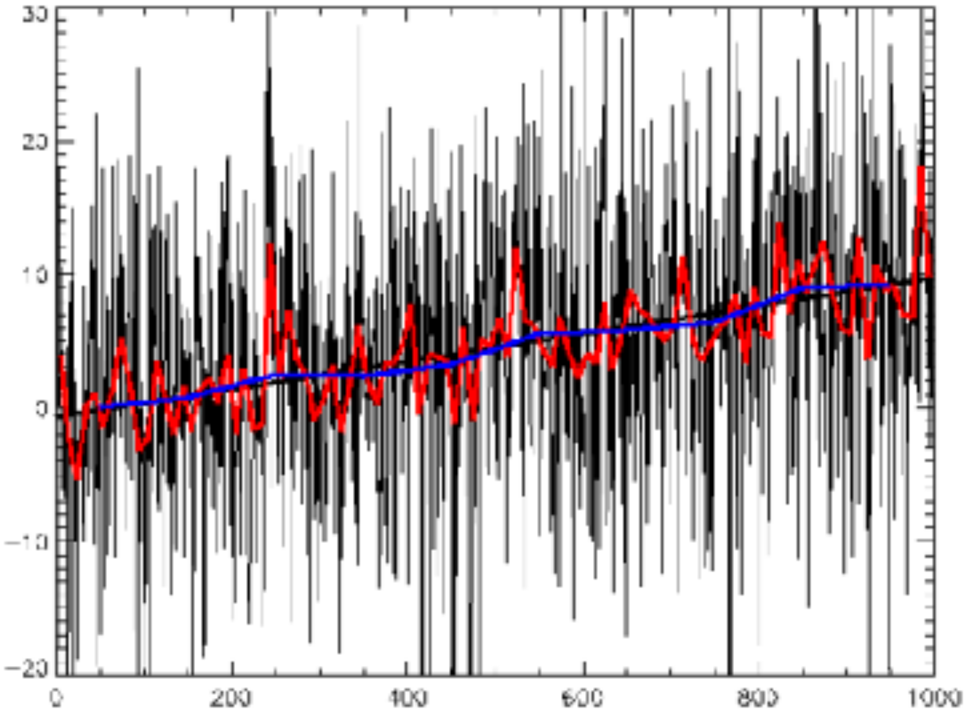
Exponential Smoothing

- MA gives equal weight to all items in window
- Exp Smoothing assigns exponentially decreasing weights for older items



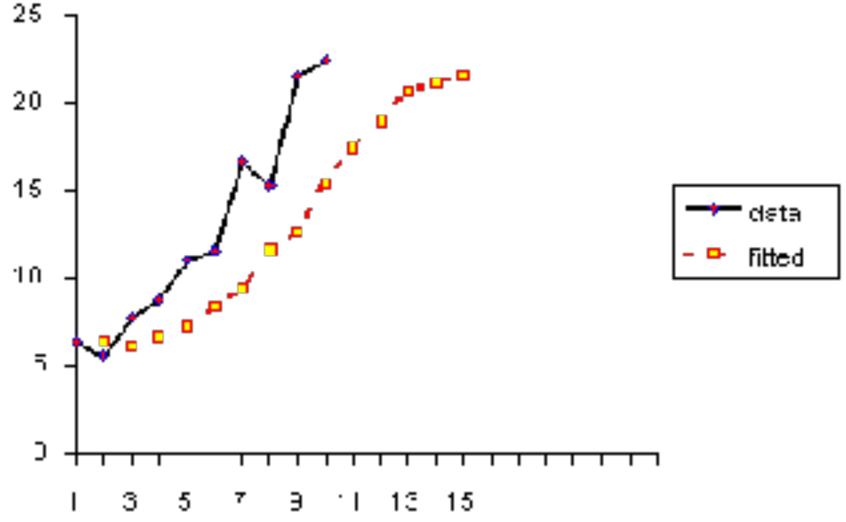
Smoothing/Averaging/Filtering

- Need to remove natural variations in data
- Shows trends unhindered by local variations



Smoothing and Forecasting

- Exponential Smoothing
  - $(y_1, y_2, \ldots, y_n)$ = sequence of observations
  - $(S_1, S_2, \ldots, S_n)$ = smoothed observations
  - $S_t = \alpha\, y_{t-1} + (1-\alpha)S_{t-1}$
- Forecasting
  - $S_{t+1} = S_t + \alpha \underline{\epsilon}_t$
  - where $\underline{\epsilon}_t$ is the forecast error
- More complex forecasting



Analysis

- Check for stationarity
- Check for Trends (seasonality)
- Check for non-constant variance
  - Trends in transformed data
    - Log transform
- Check for Randomness
  - Autocorrelation plots

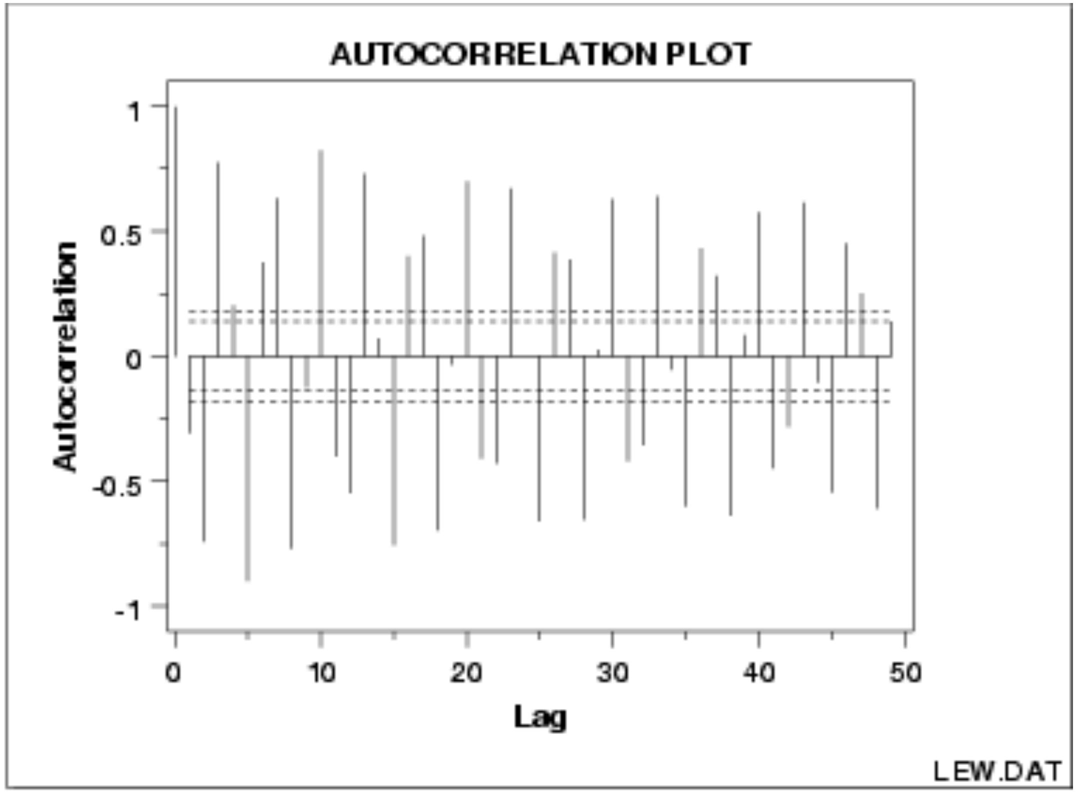
Autocorrelation: Box-Jenkins

- Attempt to find a regression connecting X_t with one or more prior values
  - Write down X_t as a linear combination of X_{t-1}, X_{t-2}, …, X_{t-p} with additive "white noise" and mean
- R-code available

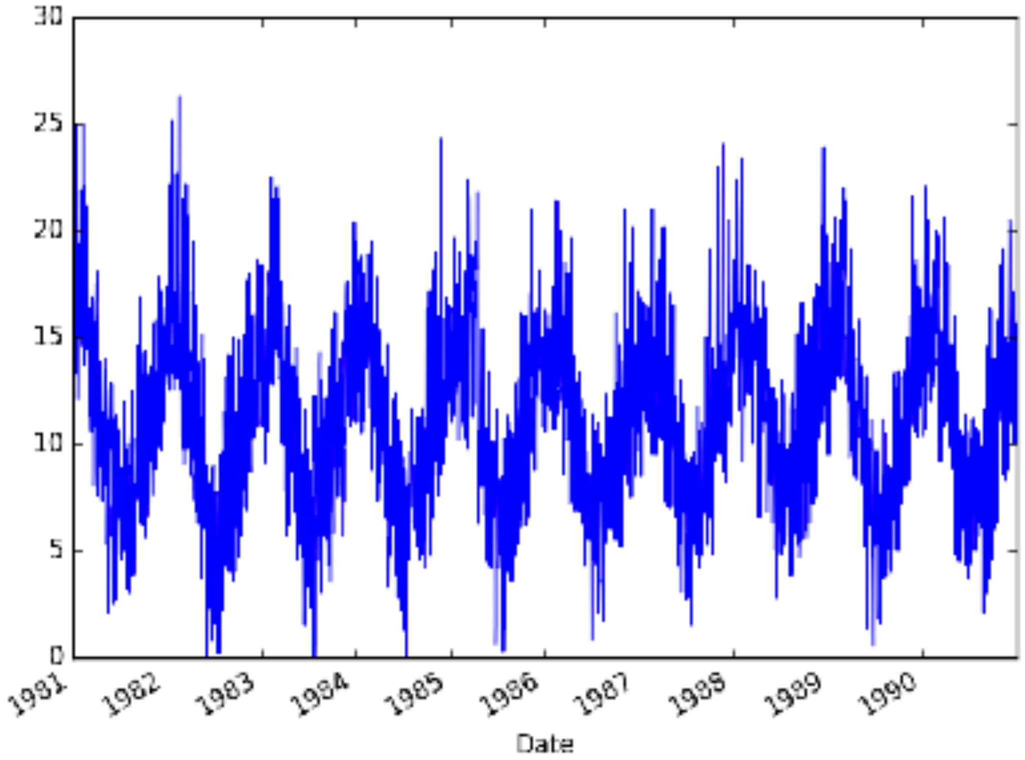
Autocorrelations

- Are the data random?
- Is an observation related to an adjacent observation? observation twice-removed? (etc.)
- Is the observed time series white noise?
- Is the observed time series sinusoidal?
- Is the observed time series autoregressive?
- What is an appropriate model for the observed time series?
- Is the model: Y = constant + error valid and sufficient? [Random data]

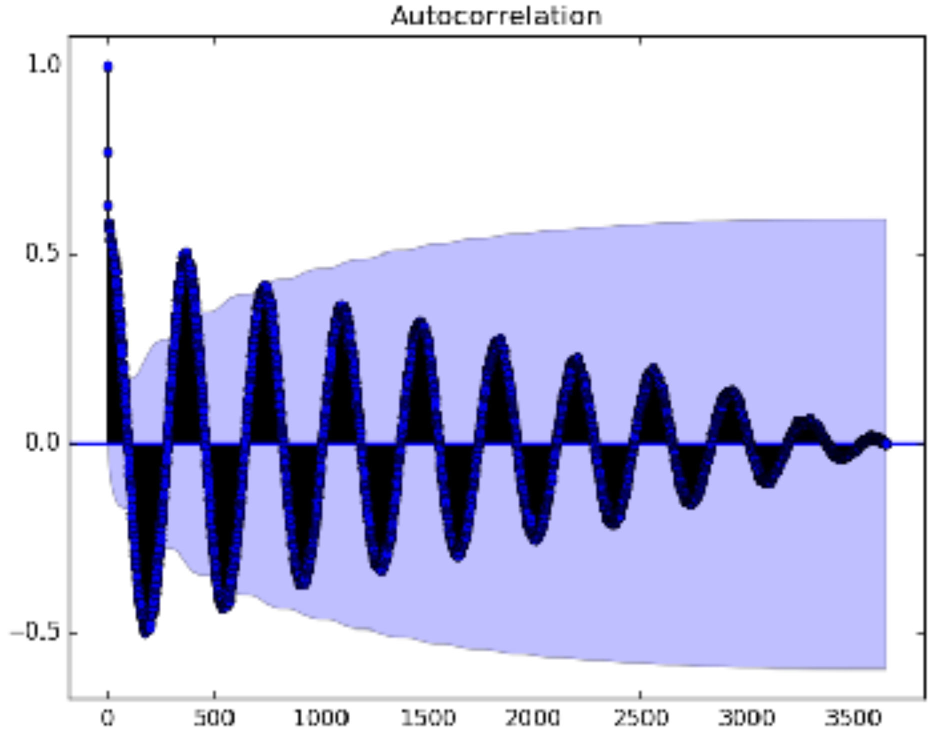
Sinusoidal curves



- The data come from an underlying sinusoidal model.
- Note alternating sequence of positive and negative spikes.
- Spikes are not decaying.

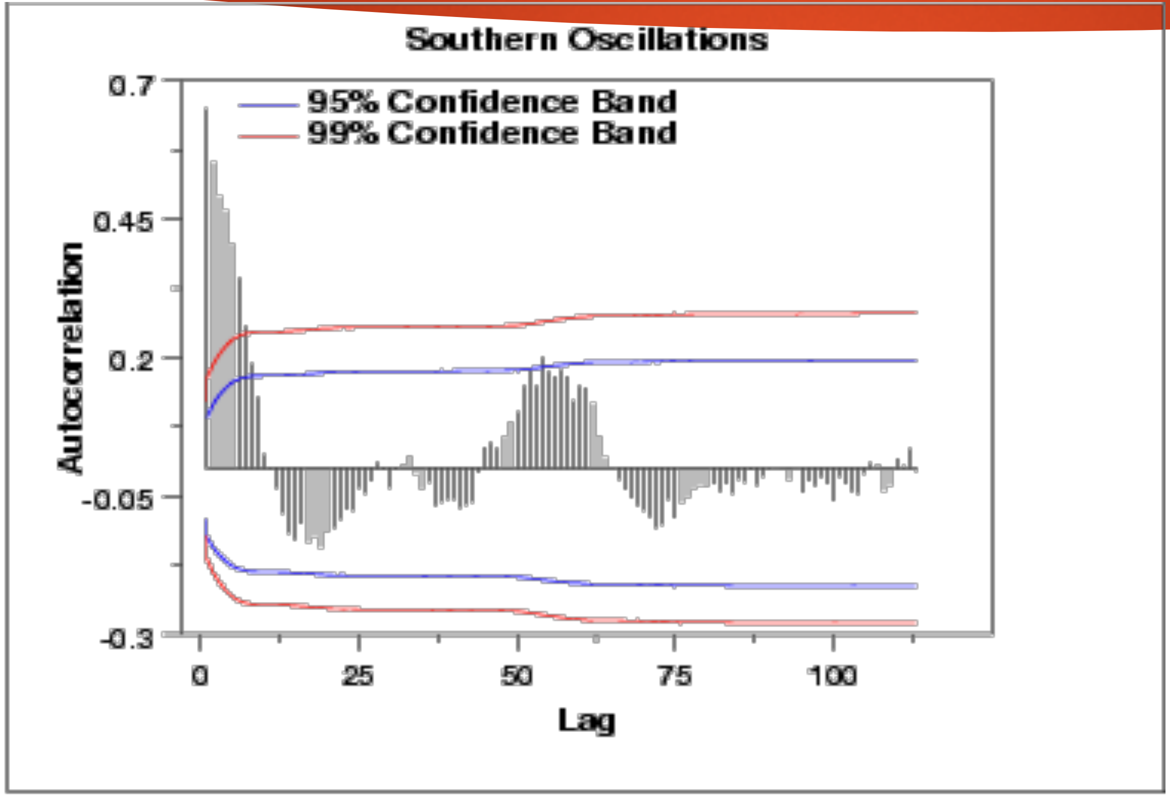


30
25
20
15
10
5
0
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
Date



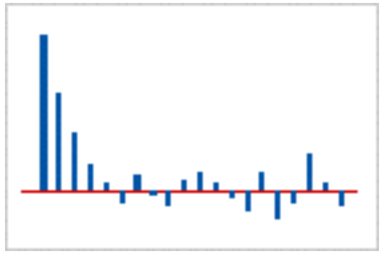
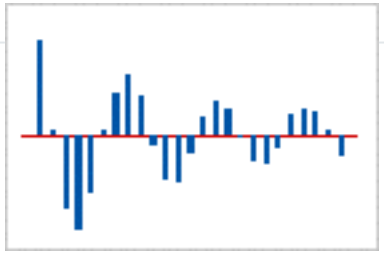
Autocorrelation
1.0
0.5
0.0
−0.5
0
500
1000
1500
2000
2500
3000
3500

Autocorrelation Plots



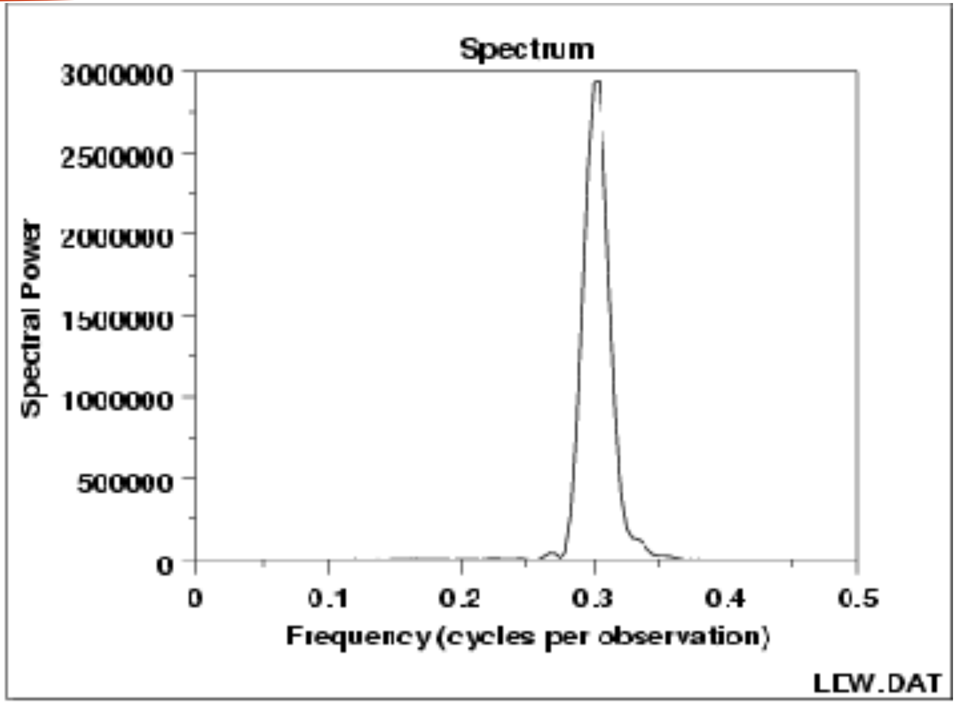
- Plot shows mixture of
  - exponentially decaying
  - damped sinusoidals
- Need autoregressive model with order > 1
- Partial autocorrelation plot needed

Interpreting patterns

Pattern	What the pattern indicates	Example
Large spike at lag 1 that decreases after a few lags.	An autoregressive term in the data. Use the partial autocorrelation function to determine the order of the autoregressive term.	
Large spike at lag 1 followed by a decreasing wave that alternates between positive and negative correlations.	A higher order autoregressive term in the data. Use the partial autocorrelation function to determine the order of the autoregressive term.	
Significant correlations at the first or second lag, followed by correlations that are not significant.	A moving average term in the data. The number of significant correlations indicates the order of the moving average term.	

Spectral Plots

- Useful to analyze plots with complex cyclical structures



Outliers

FROM JOHNSON & WICHERN, *APPLIED MULTIVARIATE STATISTICAL ANALYSIS*, 6TH ED

Canadian Hockey

- Kids trained early; Leagues for age groups
- Most talented get on Major Jr A league team& compete for Memorial Cup
- What makes a top-notch hockey player?
- Soccer, Baseball, Cricket, Swimming, Gymnastics

matured. We all know that successful people come from hardy seeds. But do we know enough about the sunlight that warmed them, the soil in which they put down the roots, and the rabbits and lumberjacks they were lucky enough to avoid? This is not a book about tall trees. It's a book about forests—and hockey is a good place to start because the explanation for who gets to the top of the hockey world is a lot more interesting and complicated than it looks. In fact, it's downright peculiar.

3

Jan	May 8	2	Sep	
Feb	Jun 3		1	1
Mar	Jul 3		Oct	
Apr	Aug 3	2		1
			Nov	
			Dec	1

No.	Name	Pos.	L/R	Height	Weight	Birth Date	Hometown
22	Tyler Ennis	C	L	5'9"	160	Oct. 6, 1989	Edmonton, AB
23	Jordan Hickmott	C	R	6'	183	Apr. 11, 1990	Mission, BC
25	Jakub Rumpel	RW	R	5'8"	166	Jan. 27, 1987	Hrnciarovce, SLO
28	Bretton Cameron	C	R	5'11"	168	Jan. 26, 1989	Didsbury, AB
36	Chris Stevens	LW	L	5'10"	197	Aug. 20, 1986	Dawson Creek, BC
3	Gord Baldwin	D	L	6'5"	205	Mar. 1, 1987	Winnipeg, MB
4	David Schlemko	D	L	6'1"	195	May 7, 1987	Edmonton, AB
5	Trever Glass	D	L	6'	190	Jan. 22, 1988	Cochrane, AB
10	Kris Russell	D	L	5'10"	177	May 2, 1987	Caroline, AB
18	Michael Sauer	D	R	6'3"	205	Aug. 7, 1987	Sartell, MN
24	Mark Isherwood	D	R	6'	183	Jan. 31, 1989	Abbotsford, BC
27	Shayne Brown	D	L	6'1"	198	Feb. 20, 1989	Stony Plain, AB
29	Jordan Bendfeld	D	R	6'3"	230	Feb. 9, 1988	Leduc, AB
31	Ryan Holfeld	G	L	5'11"	166	Jun. 29, 1989	LeRoy, SK
33	Matt Keetley	G	R	6'2"	189	Apr. 27, 1986	Medicine Hat, AB

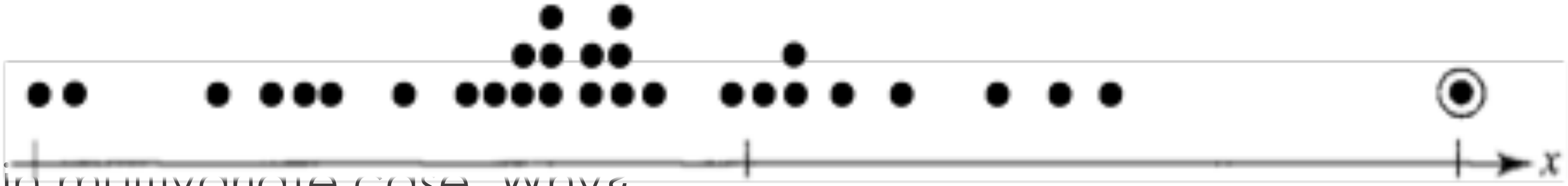
No.	Player	Birth Date	Position
1	Marcel Gecov	Jan. 1, 1988	MF
2	Ludek Frydrych	Jan. 3, 1987	GK
3	Petr Janda	Jan. 5, 1987	MF
4	Jakub Dohnalek	Jan. 12, 1988	DF
5	Jakub Mares	Jan. 26, 1987	MF
6	Michal Held	Jan. 27, 1987	DF
7	Marek Strestik	Feb. 1, 1987	FW
8	Jiri Valenta	Feb. 14, 1988	MF
9	Jan Simunek	Feb. 20, 1987	DF
10	Tomas Oklestek	Feb. 21, 1987	MF
11	Lubos Kalouda	Feb. 21, 1987	MF
12	Radek Petr	Feb. 24, 1987	GK
13	Ondrej Mazuch	Mar. 15, 1989	DF
14	Ondrej Kudela	Mar. 26, 1987	MF
15	Marek Suchy	Mar. 29, 1988	DF
16	Martin Fenin	Apr. 16, 1987	FW
17	Tomas Pekhart	May 26, 1989	FW
18	Lukas Kuban	Jun. 22, 1987	DF
19	Tomas Cihlar	Jun. 24, 1987	DF
20	Tomas Frystak	Aug. 18, 1987	GK
21	Tomas Micola	Sep. 26, 1988	MF

Canadian Hockey Players

- Cutoff birthdate is the key
- Only accept kids who are not yet 10 on Jan 1
- January Kids matured almost one extra year over December kids

Detecting Outliers

- Visual detection



- Harder in multivariate case. Why?
  - May be univariate or multivariate outlier

Bivariate Outliers

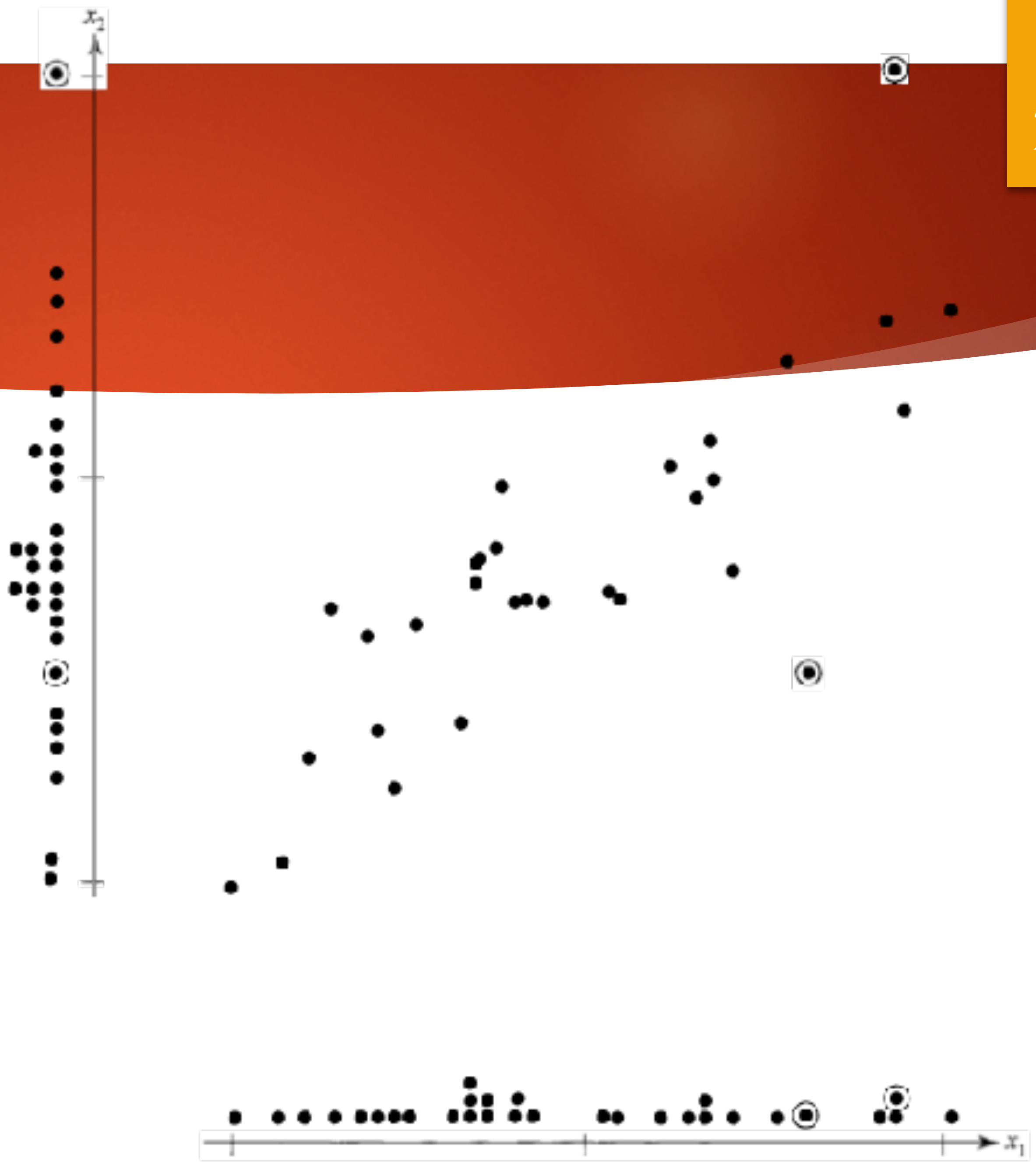


Figure 4.10 Two outliers; one univariate and one bivariate.

Multivariate Outliers

- Some outliers are hard to detect
- Look for large values of
  - $$(\mathbf{x}_j - \bar{\mathbf{x}})'\mathbf{S}^{-1}(\mathbf{x}_j - \bar{\mathbf{x}}).$$

$$
\mathbf{X} = \begin{bmatrix}
x_{11} & x_{12} & \cdots & x_{1k} & \cdots & x_{1p} \\
x_{21} & x_{22} & \cdots & x_{2k} & \cdots & x_{2p} \\
\vdots & \vdots & & \vdots & & \vdots \\
x_{j1} & x_{j2} & \cdots & x_{jk} & \cdots & x_{jp} \\
\vdots & \vdots & & \vdots & & \vdots \\
x_{n1} & x_{n2} & \cdots & x_{nk} & \cdots & x_{np}
\end{bmatrix}
$$

- p variables
- n items/samples

Sample Covariance & Correlation

The *sample covariance*

$$s_{ik} = \frac{1}{n}\sum_{j=1}^{n}(x_{ji} - \bar{x}_i)(x_{jk} - \bar{x}_k) \qquad i = 1, 2, \ldots, p, \quad k = 1, 2, \ldots, p$$

The sample correlation coefficient for the ith and kth variables is defined as

$$r_{ik} = \frac{s_{ik}}{\sqrt{s_{ii}}\sqrt{s_{kk}}} = \frac{\sum_{j=1}^{n}(x_{ji} - \bar{x}_i)(x_{jk} - \bar{x}_k)}{\sqrt{\sum_{j=1}^{n}(x_{ji} - \bar{x}_i)^2}\sqrt{\sum_{j=1}^{n}(x_{jk} - \bar{x}_k)^2}}$$

Basic Descriptive Statistics

Sample means

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{bmatrix}$$

Sample variances and covariances

$$\mathbf{S}_n = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1p} \\ s_{21} & s_{22} & \cdots & s_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ s_{p1} & s_{p2} & \cdots & s_{pp} \end{bmatrix}$$

Sample correlations

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1p} \\ r_{21} & 1 & \cdots & r_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ r_{p1} & r_{p2} & \cdots & 1 \end{bmatrix}$$

Outlier detection

- Dot plots for each variable
- Scatter plot for each pair of variables
- Calculate z-values and examine for outliers

$$z_{jk} = (x_{jk} - \bar{x}_k)/\sqrt{s_{kk}}$$

- Calculate gen sq distances & look for outliers

$$(\mathbf{x}_j - \bar{\mathbf{x}})'\mathbf{S}^{-1}(\mathbf{x}_j - \bar{\mathbf{x}}).$$

Spotting outliers from z-values

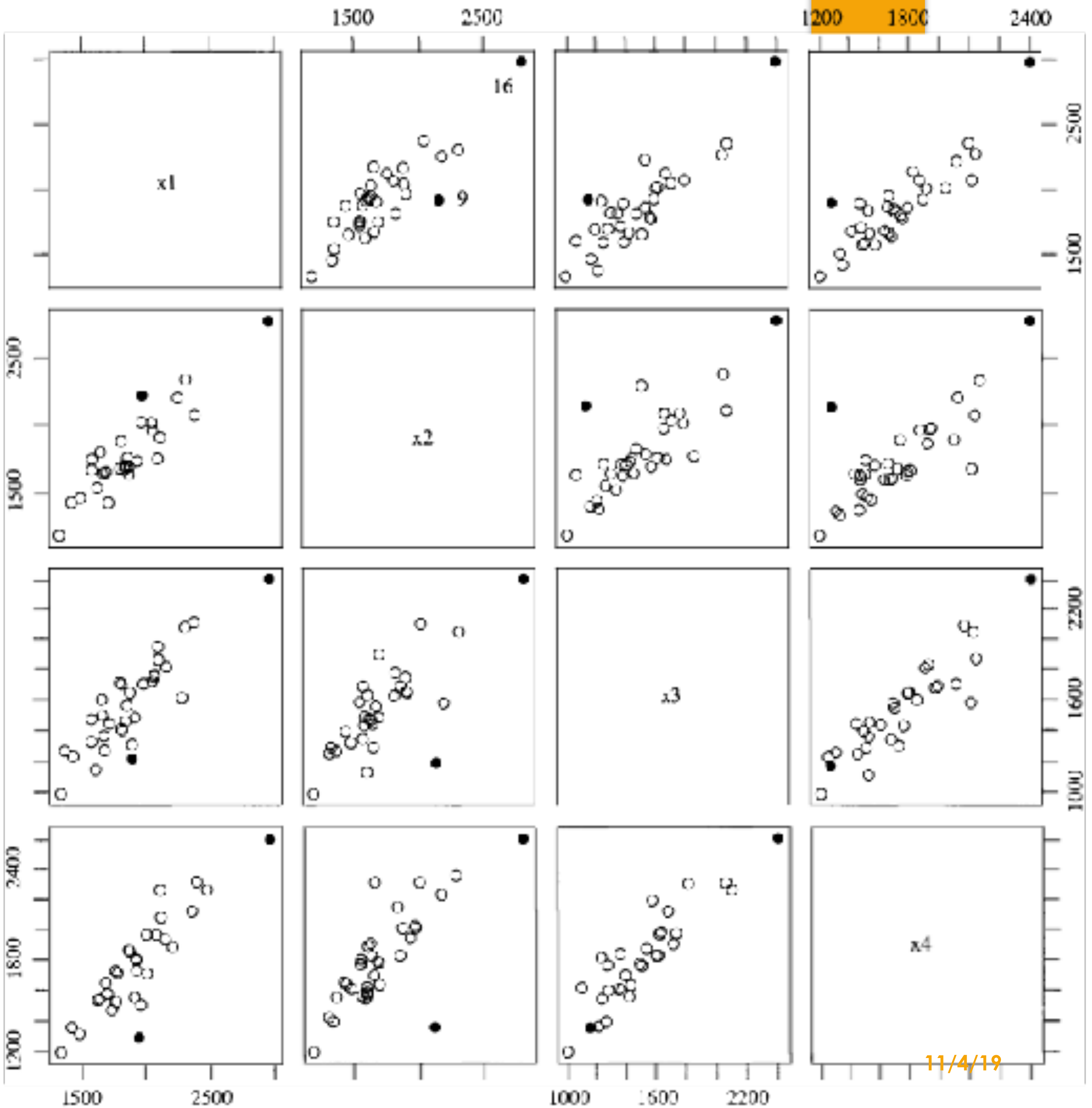
x_1	x_2	x_3	x_4	x_5	z_1	z_2	z_3	z_4	z_5
⋮	⋮	⋮	⋮	⋮ ⋮	⋮	⋮	⋮	⋮	
1631	1528	1452	1559	1602	.06	−.15	.05	.28	−.12
1770	1677	1707	1738	1785	.64	.43	1.07	.94	.60
1376	1190	723	1285	2791	−1.01	−1.47	−2.87	−.73	4.57
1705	1577	1332	1703	1664	.37	.04	−.43	.81	.13
1643	1535	1510	1494	1582	.11	−.12	.28	.04	−.20
1567	1510	1301	1405	1553	−.21	−.22	−.56	−.28	−.31
1528	1591	1714	1685	1698	−.38	.10	1.10	.75	.26
1803	1826	1748	2746	1764	.78	1.01	1.23	4.65	.52
1587	1554	1352	1554	1551	−.13	−.05	−.35	.26	−.32
⋮	⋮	⋮	⋮	⋮ ⋮	⋮	⋮	⋮	⋮	

Spotting outliers from Gen. Distance values

TABLE 4.4 FOUR MEASUREMENTS OF STIFFNESS WITH STANDARDIZED VALUES

x_1	x_2	x_3	x_4	Observation no.	z_1	z_2	z_3	z_4	d^2
1889	1651	1561	1778	1	−.1	−.3	.2	.2	.60
2403	2048	2087	2197	2	1.5	.9	1.9	1.5	5.48
2119	1700	1815	2222	3	.7	−.2	1.0	1.5	7.62
1645	1627	1110	1533	4	−.8	−.4	−1.3	−.6	5.21
1976	1916	1614	1883	5	.2	.5	.3	.5	1.40
1712	1712	1439	1546	6	−.6	−.1	−.2	−.6	2.22
1943	1685	1271	1671	7	.1	−.2	−.8	−.2	4.99
2104	1820	1717	1874	8	.6	.2	.7	.5	1.49
2983	2794	2412	2581	9	3.3	3.3	3.0	2.7	12.26
1745	1600	1384	1508	10	−.5	−.5	−.4	−.7	.77
1710	1591	1518	1667	11	−.6	−.5	.0	−.2	1.93
2046	1907	1627	1898	12	.4	.5	.4	.5	.46
1840	1841	1595	1741	13	−.2	.3	.3	.0	2.70
1867	1685	1493	1678	14	−.1	−.2	−.1	−.1	.13
1859	1649	1389	1714	15	−.1	−.3	−.4	−.0	1.08
1954	2149	1180	1281	16	.1	1.3	−1.1	−1.4	16.85
1325	1170	1002	1176	17	−1.8	−1.8	−1.7	−1.7	3.50
1419	1371	1252	1308	18	−1.5	1.2	.8	−1.3	3.99
1828	1634	1602	1755	19	−.2	−.4	.3	.1	1.36
1725	1594	1313	1646	20	−.6	−.5	−.6	−.2	1.46
2276	2189	1547	2111	21	1.1	1.4	.1	1.2	9.90
1899	1614	1422	1477	22	−.0	−.4	−.3	−.8	5.06
1633	1513	1290	1516	23	−.8	−.7	−.7	−.6	.80
2061	1867	1646	2037	24	.5	.4	.5	1.0	2.54
1856	1493	1356	1533	25	−.2	−.8	−.5	−.6	4.58
1727	1412	1238	1469	26	−.6	−1.1	−.9	−.8	3.40
2168	1896	1701	1834	27	.8	.5	.6	.3	2.38
1655	1675	1414	1597	28	−.8	−.2	−.3	−.4	3.00
2326	2301	2065	2234	29	1.3	1.7	1.8	1.6	6.28
1490	1382	1214	1284	30	−1.3	−1.2	−1.0	−1.4	2.58

Harder to spot them on scatter plots!



Other Transforms for Normality

HELPFUL TRANSFORMATIONS TO NEAR NORMALITY

Original Scale	*Transformed Scale*	
1. Counts, y	$\sqrt{y}$	
2. Proportions, $\hat{p}$	$\text{logit}(\hat{p}) = \frac{1}{2}\log\left(\frac{\hat{p}}{1-\hat{p}}\right)$	(4-33)
3. Correlations, r	Fisher's $z(r) = \frac{1}{2}\log\left(\frac{1+r}{1-r}\right)$	

TABLE 1.9 NATIONAL TRACK RECORDS FOR WOMEN

Country	100 m (s)	200 m (s)	400 m (s)	800 m (min)	1500 m (min)	3000 m (min)	Marathon (min)
Argentina	11.61	22.94	54.50	2.15	4.43	9.79	178.52
Australia	11.20	22.35	51.08	1.98	4.13	9.08	152.37
Austria	11.43	23.09	50.62	1.99	4.22	9.34	159.37
Belgium	11.41	23.04	52.00	2.00	4.14	8.88	157.85
Bermuda	11.46	23.05	53.30	2.16	4.58	9.81	169.98
Brazil	11.31	23.17	52.80	2.10	4.49	9.77	168.75
Burma	12.14	24.47	55.00	2.18	4.45	9.51	191.02
Canada	11.00	22.25	50.06	2.00	4.06	8.81	149.45
Chile	12.00	24.52	54.90	2.05	4.23	9.37	171.38
China	11.95	24.41	54.97	2.08	4.33	9.31	168.48
Colombia	11.60	24.00	53.26	2.11	4.35	9.46	165.42
Cook Islands	12.90	27.10	60.40	2.30	4.84	11.10	233.22
Costa Rica	11.96	24.60	58.25	2.21	4.68	10.43	171.80
Czechoslovakia	11.09	21.97	47.99	1.89	4.14	8.92	158.85
Denmark	11.42	23.52	53.60	2.03	4.18	8.71	151.75
Dominican Republic	11.79	24.05	56.06	2.24	4.74	9.89	203.88
Finland	11.13	22.39	50.14	2.03	4.10	8.92	154.23
France	11.15	22.59	51.73	2.00	4.14	8.98	155.27
German Democratic Republic	10.81	21.71	48.16	1.93	3.96	8.75	157.68
Federal Republic of Germany	11.01	22.39	49.75	1.95	4.03	8.59	148.53
Great Britain and Northern Ireland	11.00	22.13	50.46	1.98	4.03	8.62	149.72
Greece	11.79	24.08	54.93	2.07	4.35	9.87	182.20
Guatemala	11.84	24.54	56.09	2.28	4.86	10.54	215.08
Hungary	11.45	23.06	51.50	2.01	4.14	8.98	156.37
India	11.95	24.28	53.60	2.10	4.32	9.98	188.03
Indonesia	11.85	24.24	55.34	2.22	4.61	10.02	201.28
Ireland	11.43	23.51	53.24	2.05	4.11	8.89	149.38
Israel	11.45	23.57	54.90	2.10	4.25	9.37	160.48
Italy	11.29	23.00	52.01	1.96	3.98	8.63	151.82
Japan	11.73	24.00	53.73	2.09	4.35	9.20	150.50
Kenya	11.73	23.88	52.70	2.00	4.15	9.20	181.05
Korea	11.96	24.49	55.70	2.15	4.42	9.62	164.65
Democratic People's Republic of Korea	12.25	25.78	51.20	1.97	4.25	9.35	179.17
Luxembourg	12.03	24.96	56.10	2.07	4.38	9.64	174.68
Malaysia	12.23	24.21	55.09	2.19	4.69	10.46	182.17
Mauritius	11.76	25.08	58.10	2.27	4.79	10.90	261.13
Mexico	11.89	23.62	53.76	2.04	4.25	9.59	158.53
Netherlands	11.25	22.81	52.38	1.99	4.06	9.01	152.48
New Zealand	11.55	23.13	51.60	2.02	4.18	8.76	145.48
Norway	11.58	23.31	53.12	2.03	4.01	8.53	145.48
Papua New Guinea	12.25	25.07	56.96	2.24	4.84	10.69	233.00
Philippines	11.76	23.54	54.60	2.19	4.60	10.16	200.37
Poland	11.13	22.21	49.29	1.95	3.99	8.97	160.82
Portugal	11.81	24.22	54.30	2.09	4.16	8.84	151.20
Rumania	11.44	23.46	51.20	1.92	3.96	8.53	165.45
Singapore	12.30	25.00	55.08	2.12	4.52	9.94	182.77
Spain	11.80	23.98	53.59	2.05	4.14	9.02	162.60
Sweden	11.16	22.82	51.79	2.02	4.12	8.84	154.48
Switzerland	11.45	23.31	53.11	2.02	4.07	8.77	153.42
Taiwan	11.22	22.62	52.50	2.10	4.38	9.63	177.87
Thailand	11.75	24.46	55.80	2.20	4.72	10.28	168.45
Turkey	11.98	24.44	56.45	2.15	4.37	9.38	201.08
U.S.A.	10.79	21.83	50.62	1.96	3.95	8.50	142.72
U.S.S.R.	11.06	22.19	49.19	1.89	3.87	8.45	151.22
Western Samoa	12.74	25.85	58.73	2.33	5.81	13.04	306.00

TABLE 1.9 NATIONAL TRACK RECORDS FOR WOMEN

Source: *IAAF/ATFS Track and Field Statistics Handbook for the 1984 Los Angeles Olympics.*