

FALL 2019: **COT 6405** ANALYSIS OF ALGORITHMS
[HOMEWORK 3; DUE OCT 6 BY 11:59 PM VIA CANVAS]

General submission guidelines and policies: ADD THE FOLLOWING SIGNED STATEMENT. Without this statement, your homework will not be graded.

I HAVE ADHERED TO THE COLLABORATION POLICY FOR THIS CLASS. IN OTHER WORDS, EVERYTHING WRITTEN DOWN IN THIS SUBMISSION IS MY OWN WORK. FOR PROBLEMS WHERE I RECEIVED ANY HELP, I HAVE CITED THE SOURCE, AND/OR NAMED THE COLLABORATOR.

Read the handout on **Homework guidelines and collaboration policy** from your course website before you start on this homework. This is very important. You only need to submit solutions to problems marked (**Regular**). All others are optional.

Problems

14. (**Regular**) Prove that finding the minimum (or maximum) of n numbers has a lower bound of $\Omega(n)$ for any comparison-based algorithm.
15. (**Exercise**) Modify the basic version of QUICKSORT to run in worst-case $O(n \log n)$ time by using the median as the pivot on every recursive call.
16. (**Regular**) Given an array A with n items indexed from 1 through n , describe how you can efficiently answer any query of the form FINDMAX(i, j), which returns the largest item in A between indices i and j . If there are duplicates, your query may return any one of them. Carefully describe any data structure you use. If you are modifying an algorithm from the book in simple ways, then you may simply explain the modifications without having to reproduce the whole algorithm.
17. (**Extra Credit**) Given a sorted array A with n items, many of which are repeated multiple times, describe how you can efficiently answer any query of the form MOSTFREQUENT(i, j), which returns the most frequent item in A between indices i and j . If there is more than one possible answer, your query may return any one of them. Carefully describe any data structure you use. If you are modifying an algorithm from the book in simple ways, then you may simply explain the modifications without having to reproduce the whole algorithm.
18. (**Exercise**) Solve 16.3-3, page 436.
19. (**Exercise**) Study Section 15.2 from the book and solve 15.1-5, page 370.
20. (**Regular**) Becky Thompson wants to cut a wooden board of length L (and standard width), which needs to be cut into n smaller pieces for her shop. She has been asked to make the cuts at locations l_1, l_2, \dots, l_n ft from the left end of the board. However, Tom

Sawyer who cuts the boards for her charges money for cutting. Tom's cutting rates are strange; if he cuts a board of length x into two smaller pieces (of any lengths), he will charge $\$x$.

The cutting order will determine the cost of the cuttings required. For example, assume that the board is of length 100 ft and that Becky needs cuts at locations 20ft, 40ft and 70ft from the left end. If Tom cuts it in that order, then Becky's cost will be $\$100 + 80 + 60 = \240 . On the other hand, if Tom cuts it at 40ft first and then at 20ft and 70ft for the two smaller pieces, then the total cost will be only $\$100 + 40 + 60 = \200 .

Design an algorithm to determine the optimal order of cuts required to minimize Beck's sawing costs. Analyze your algorithm.