Celebrity Problem

- A **Celebrity** is one that knows nobody and that everybody knows.

Celebrity Problem:

INPUT: n persons with a $n \times n$ information matrix.

OUTPUT: Find the "celebrity", if one exists.

MODEL: Only allowable questions are:

  - *Does person* i *know person* j*?*

- Naive Algorithm: $O(n^2)$ Questions.

Celebrity Problem (Cont'd)

- **Induction Hypothesis:** We know how to find a celebrity (if one exists) among a set of $n-1$ people.

[The above hypothesis leads to inefficient solution.]

Given n persons, 3 cases arise:

1. Celebrity is among the first $n-1$persons
2. Celebrity is the n-th person.
3. No celebrity exists.

Celebrity Problem (Cont'd)

- **Induction Hypothesis 2:** We know how to find n-2 non-celebrities among a set of n-1 people, i.e., we know how to find at most one person among a set of n-1 people that could potentially be a celebrity.

- Resulting algorithm needs [3(n-1)-1] questions.

Psychic Assist Hotline

- Ms. Cleo gives me 15 numbers and promises me that at least 4 will appear in Saturday’s FL Lottery.
- How many tickets do I need to buy to guarantee at least one ticket to have 3 correct numbers?

Smaller Problem

- Suppose Ms. Cleo gives me 6 numbers from which 4 are guaranteed. Every ticket has 3 numbers and I need 3 to win.
- Cover all 3-sets.
- Suppose I pick {1,2,3}, {1,2,4} & {1,2,5}. Then should I also have picked {1,2,6}?
- NO!

Psychic Assist Hotline

- Ms. Cleo gives me 15 numbers and promises me that at least 4 will appear in Saturday's FL Lottery.
- How many tickets do I need to buy to guarantee at least one ticket with at least 3 correct numbers?
- FIVE!!! (if you assume that numbers come from 1 through 44).

Psychic Problem

- Initialize all k-sets as “uncovered”.
- While (there is a “uncovered” k-set)
  - Select a ticket that contains it
  - Update the set of “covered” k-sets.