Psychic Assist Hotline

- Ms. Cleo gives me 15 numbers and promises me that at least 4 will appear in Saturday’s FL Lottery.
- How many tickets do I need to buy to guarantee at least one ticket with at least 3 correct numbers?

- FIVE!!! (if you assume that numbers come from 1 through 44).
Psychic Problem

- Initialize all k-sets as “uncovered”.
- While (there is a “uncovered” k-set)
  - Select a ticket that contains it
  - Update the set of “covered” k-sets.
Evolution of Data Structures

• Complex problems require complex data structures.
• Simple data types → Lists.
• Applications of lists include: students roster, list of voters, grocery list, list of transactions, etc.
• Array implementation of list: random access.
• Need for list “operations” arose – “Static” vs. “dynamic” lists. “Storing” items in list vs. “Maintaining” items in list.
• Lot of research on “Sorting” and “Searching”.
• “Inserting” in a specified location in a list caused the following evolution: Array implementation → Linked list implementation.
• Other linear structures e.g., stacks, queues, etc.
Evolution of Data Structures

- Trees made hierarchical organization of data easy to handle. Applications of trees: administrative hierarchy in a business setup, storing an arithmetic expression, organization of the functions calls of a recursive program, etc.
- Search trees (e.g., BST) were designed to make search and retrieval efficient in trees. A BST may not allow fast search or retrieval, if it is very unbalanced, since the time complexities of the operations depended on the height of the tree.
- Graphs generalize trees; model more general networks.
- Abstract data types. Advantages include: Encapsulation of data and operations, hiding of unnecessary details, localization and debugging of errors, ease of use since interface is clearly specified, ease of program development, etc.
Solving Recurrence Relations

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<table>
<thead>
<tr>
<th>Recurrence; Cond</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(n) = T(n-1) + O(1)$</td>
<td>$T(n) = O(n)$</td>
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<tr>
<td>$T(n) = T(n-1) + O(n)$</td>
<td>$T(n) = O(n^2)$</td>
</tr>
<tr>
<td>$T(n) = T(n-c) + O(1)$</td>
<td>$T(n) = O(n)$</td>
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<tr>
<td>$T(n) = T(n-c) + O(n)$</td>
<td>$T(n) = O(n^2)$</td>
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<tr>
<td>$T(n) = 2T(n/2) + O(n)$</td>
<td>$T(n) = O(n \log n)$</td>
</tr>
<tr>
<td>$T(n) = aT(n/b) + O(n)$; $a = b$</td>
<td>$T(n) = O(n \log n)$</td>
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<tr>
<td>$T(n) = aT(n/b) + O(n)$; $a &lt; b$</td>
<td>$T(n) = O(n)$</td>
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<tr>
<td>$T(n) = aT(n/b) + f(n)$; $f(n) = O(n^{\log_b a-\epsilon})$</td>
<td>$T(n) = O(n)$</td>
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<tr>
<td>$T(n) = aT(n/b) + f(n)$; $f(n) = O(n^{\log_b a})$</td>
<td>$T(n) = \Theta(n^{\log_b a} \log n)$</td>
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<tr>
<td>$T(n) = aT(n/b) + f(n)$; $af(n/b) \leq cf(n)$</td>
<td>$T(n) = \Omega(n^{\log_b a} \log n)$</td>
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</table>
Sorting Algorithms

- Selection Sort
- Insertion Sort
- Bubble Sort
- Shaker Sort
- Shell Sort
- Merge Sort
- Heap Sort
- Quick Sort
- Bucket & Radix Sort
- Counting Sort
Algorithm Invariants

- **Selection Sort**
  - iteration k: the k smallest items are in correct location.

- **Insertion Sort**
  - iteration k: the first k items are in sorted order.

- **Bubble Sort**
  - In each pass, every item that does not have a smaller item after it, is moved as far up in the list as possible.
  - Iteration k: k smallest items are in the correct location.

- **Shaker Sort**
  - In each odd (even) numbered pass, every item that does not have a smaller (larger) item after it, is moved as far up (down) in the list as possible.
  - Iteration k: the k/2 smallest and largest items are in the correct location.
Algorithm Invariants (Cont’d)

- Merge (many lists)
  - Iteration k: the k smallest items from the lists are merged.

- Heapify
  - Iteration with i = k: Subtrees with roots at indices k or larger satisfy the heap property.

- HeapSort
  - Iteration k: Largest k items are in the right location.

- Partition (two sublists)
  - Iteration k (with pointers at i and j): items in locations [1..I] (locations [i+1..j]) are at least as small (large) as the pivot.
Sorting Algorithms

- Number of Comparisons
- Number of Data Movements
- Additional Space Requirements
QuickSort(A, p, r)
  if (p < r) then
    q = Partition(A, p, r)
    QuickSort(A, p, q-1)
    QuickSort(A, q+1, r)

Partition(A, p, r)
  x = A[r]
  i = p-1
  for j = p to r-1 do
    if A[j] <= x) then
      i++
      exchange(A[i], A[j])
  exchange(A[i+1], A[r])
  return i+1
**Figure 8.5**
Shell sort after each pass if the increment sequence is \{1, 3, 5\}

<table>
<thead>
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<th>Original</th>
<th>81</th>
<th>94</th>
<th>11</th>
<th>96</th>
<th>12</th>
<th>35</th>
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<th>95</th>
<th>28</th>
<th>58</th>
<th>41</th>
<th>75</th>
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<tbody>
<tr>
<td>After 5-sort</td>
<td>35</td>
<td>17</td>
<td>11</td>
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<td>75</td>
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<td>58</td>
<td>81</td>
<td>94</td>
<td>95</td>
</tr>
<tr>
<td>After 3-sort</td>
<td>28</td>
<td>12</td>
<td>11</td>
<td>35</td>
<td>15</td>
<td>41</td>
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<td>94</td>
<td>75</td>
<td>81</td>
<td>96</td>
<td>95</td>
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<tr>
<td>After 1-sort</td>
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<td>15</td>
<td>17</td>
<td>28</td>
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<td>41</td>
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<td>75</td>
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<td>94</td>
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<td>96</td>
</tr>
</tbody>
</table>
ShellSort

public static void shellsort( Comparable [ ] a )
{
    for( int gap = a.length / 2; gap > 0;
        gap = gap == 2 ? 1 : (int) ( gap / 2.2 ) )
    for( int i = gap; i < a.length; i++ )
    {
        Comparable tmp = a[ i ];
        int j = i;

        for( ; j >= gap && tmp.compareTo( a[ j - gap ] ) < 0; j -= gap )
            a[ j ] = a[ j - gap ];
        a[ j ] = tmp;
    }
}
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