

COT 6936: Topics in Algorithms

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[http://www.cs.fiu.edu/~giri/teach/COT6936\\_S10.html](http://www.cs.fiu.edu/~giri/teach/COT6936_S10.html)  
<https://online.cis.fiu.edu/portal/course/view.php?id=427>

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Massive Data Sets

- Examples of large persistent data sets
  - WalMart Transaction data (1 PB?)
  - Sloan Digital Sky Survey (100 TB)
  - Web (over a Trillion pages; over 1 PB of text)
  - CERN (expected to produce ~40 TB/sec)
- Large data sets with time-sensitive data
  - Financial tickers data
  - Credit Card usage traffic
  - Network Traffic: Telecom & ISP traffic
  - Sensor data

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Important Issues for Stream Algorithms

- Key parameters
  - Amount of memory available; window size
  - Per-item processing time; # of Passes on data
  - Tolerance to error
- What is needed?
  - Summarizations, synposes, sketches
  - Randomization and sampling
  - Pattern Discovery
  - Anomaly Detection
  - Clustering and Classifications

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### Streaming Model of Computation

- $N = \#$  of items seen so far, window size
  - amount of memory available
- $\epsilon =$  error tolerance
- Memory usage =  $\text{poly}(1/\epsilon, \log N)$
- Query Time =  $\text{poly}(1/\epsilon, \log N)$

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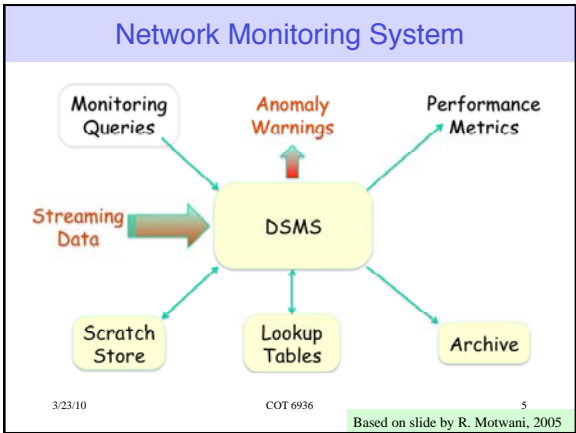
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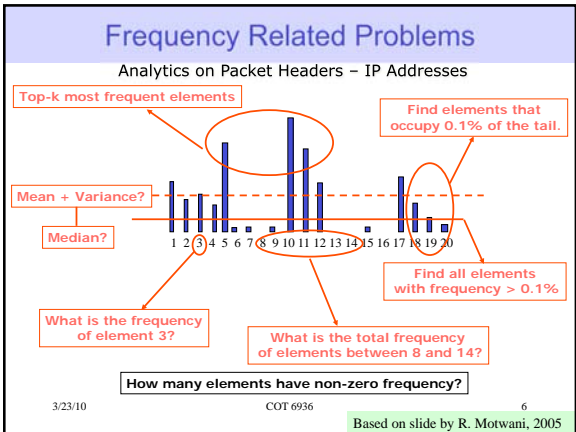
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### Warm up Problems

- Given stream of values, find **mean**.
  - Easy.
  - Maintain sum of all values and number of items
- Given stream, find **standard deviation**.
  - Not so hard
- Given stream of bits and window size  $N$ , count number of 1s in window
  - Space and time?
    - Naive: Store the window: requires  $N$  bits
    - Can you do better?

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### Problem: Finding Missing Labels

- Packets arrive in random order, each is labeled from set  $\{1, \dots, n\}$ .
- Assume that one packet is missing.
- Find the label of the missing packet.
  
- Bit vector of length  $n$ 
  - Space  $O(n)$
- Maintain sum of labels and subtract from  $N$ 
  - Space  $O(\log n)$

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### Problem: Finding Missing Numbers

- Same as problem 1, but there may be up to  $k$  missing numbers.
- Instead of sum of numbers, we maintain  $k$  different functions of the numbers seen.
  - Decoding is not so easy
    - Needs factoring polynomials
  - Randomized algorithms
    - $O(k^2 \log n)$
    - $O(k \log k \log n)$

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Problem: Find number of unique items

- Simple hashing scheme to do counting
  - Space =  $O(m)$
  - Time =  $O(1)$  per item in stream

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Problem: Find fraction of rare items

- Rarity  $r[t] = |\{j \mid c_t[j] = 1\}| / u$ 
  - Number of items in stream that are rare (i.e., appear only once)

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Problem: Counting

- Given a stream of bits, at every time instant, maintain count of number of 1s in last N elements
  - Deterministic algorithms
    - $\Theta(N)$  bits of memory to answer in  $O(1)$  time [Why?]

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**Problem: Counting**

- How well can you approximate with  $o(N)$  memory? [Datar et al. SIAM J C 2002]
  - Use histogram techniques
    - Build time-based histograms in which every bucket represents a contiguous time interval
    - Idea: Use uniform buckets
    - Problem: 1s may not be distributed uniformly
    - Solution: Use non-uniform buckets
  - Results
    - $O((1/\epsilon)\log^2 N)$  bits  $\Omega((1/\epsilon)\log^2(N\epsilon))$
    - $(1+\epsilon)$ -approximate count in  $O(1)$  time

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**Other problems**

- **COUNTING:** Given a stream of bits, at every time instant, maintain count of number of 1s in last  $N$  elements
- **SUM:** Given a stream of positive integers in range  $[0..R]$ , at every time instant, maintain sum of last  $N$  elements

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**Clustering**

- **K-Means**
  - Constant-factor approximation,  $O(nk \log k)$  time,  $O(k)$  space, single pass [Charikar et al. 1997]
- **K-Medians**
  - Constant-factor approximation,  $O(nk \log k)$  time,  $O(n^\epsilon)$  space, single pass [Guha et al. 2002]

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