COT 6936: Topics in Algorithms

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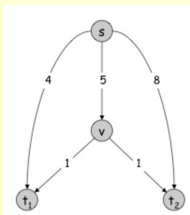
http://www.cs.fiu.edu/~giri/teach/COT6936_S10.html
https://online.cis.fiu.edu/portal/course/view.php?id=427

Nash Equilibria: Multicast Routing

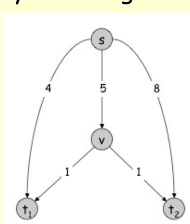
- Multicast routing: Given directed graph $G = (V, E)$ with edge costs $c_e \geq 0$, source node s, k agents located at terminal nodes $t_1, \ldots, t_k$.
  - Agent j must construct path P_j from node s to its terminal t_j.
  - Fair share. If x agents use edge e, they each pay c_e / x.



Example

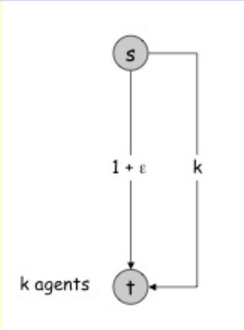
- Best response dynamics: Each agent is continually prepared to improve its solution in response to changes made by other agents

1	2	1 pays	2 pays
Outer	Outer	4	8
Outer	Middle	4	5+1
Middle	Outer	5+1	8
Middle	Middle	5/2 + 1	5/2 + 1



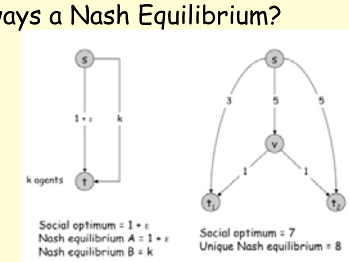
- Nash Equilibrium: When no agent has incentive to switch

Multiple Nash Equilibria



Comparing Nash Equilibria: Price of Stability

- Social Optimum: Solution that optimizes total cost to all agents
- Is it always a Nash Equilibrium?



Local Search Algorithm

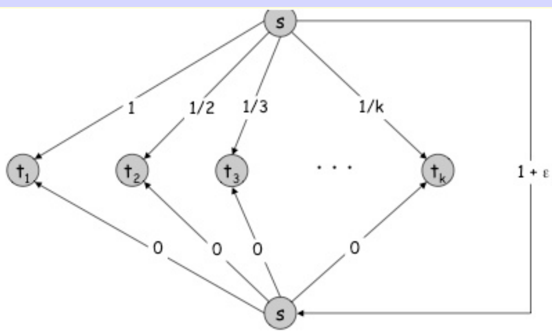
- Best response dynamics: Each agent is continually prepared to improve its solution in response to changes made by other agents
  - Best-Response-Dynamics(G, c) {

```
    Pick a path for each agent
    while (not a Nash equilibrium) {
        Pick an agent i who can improve by
            switching paths
        Switch path of agent i
    }
}
```

Local Search

- Best response dynamics: Each agent is continually prepared to improve its solution in response to changes made by other agents
  - How do we know a Nash Equilibrium exists?
  - Is there a strategy that will lead to Nash Equilibrium?
  - Does the best response dynamics strategy always result in a Nash Equilibrium?

Another example



Results

- Nash Equilibrium always exists
- Best-response dynamics always leads to a set of paths that form a Nash Equilibrium solution
- For every instance, there is a Nash Equilibrium solution for which total cost to all agents exceeds that of social optimum by at most a factor of H(k)

Random Walks

- Let G = (V,E) be an undirected graph with n vertices and m edges. Let N(v) be the neighbors of v in G.
- Random walk on G:
  - Starts at vertex v_0
  - At each step it proceeds to a randomly chosen neighbor, i.e., from vertex v proceeds to one of the vertices in N(v) with prob 1/|N(v)|

Typical questions

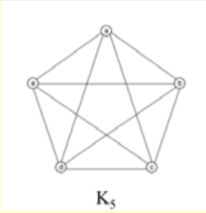
- Hitting time (First Passage time): H_{uv} = the expected number of steps to get from vertex u to vertex v
- Commute time: $C_{uv} = H_{uv} + H_{vu}$
- C_u = expected number of steps in a walk that starts at u and ends upon visiting every vertex at least once
- Cover time: $C(G) = \max_u C_u$

Chain graphs

- H_{uv} = ??
- $C(L_n)$ = ??

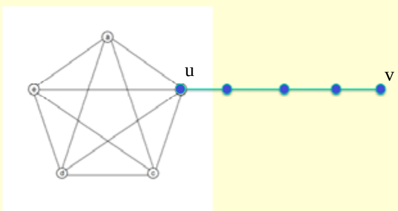
Cliques

- K_n = clique on n vertices
- H_{uv} = ??
- $C(K_n)$ = ??



Lollipop graphs

- L_n = clique on n/2 vertices with a chain of length n/2
- $H_{uv} \neq H_{vu}$



Connection to Resistive Networks

- $C_{uv} = 2\, m\, R_{uv}$