COT 6936: Topics in Algorithms

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Presentation Outline

Applications

- Graphics

Applications

- Graphics
- Robotics

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- Graphics
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- Sensor Networks

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- ...

Presentation Outline

Convex Hull Problem

- This is a fundamental problem in Computational Geometry.
- There are many algorithms for solving this problem . . .

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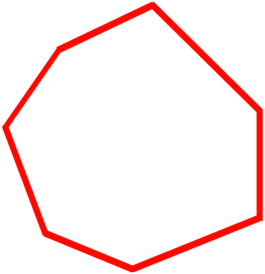

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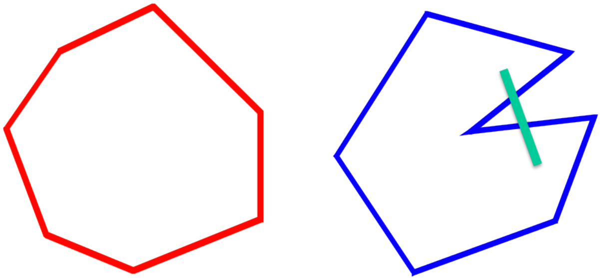

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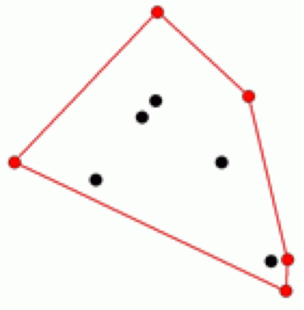

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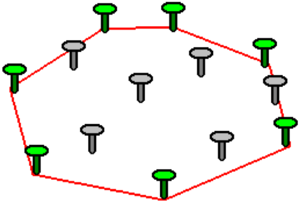

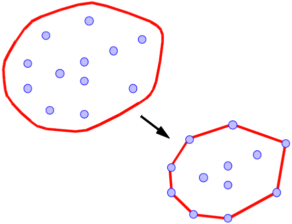

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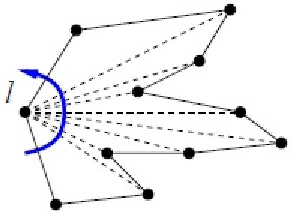

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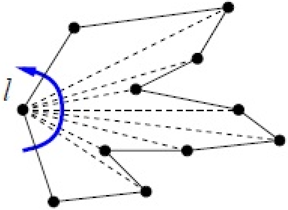

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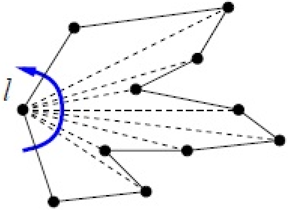

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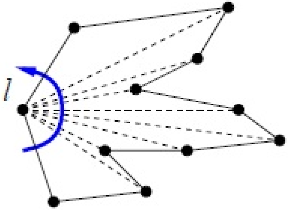

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$O(n \log n)$ time, mainly for sorting

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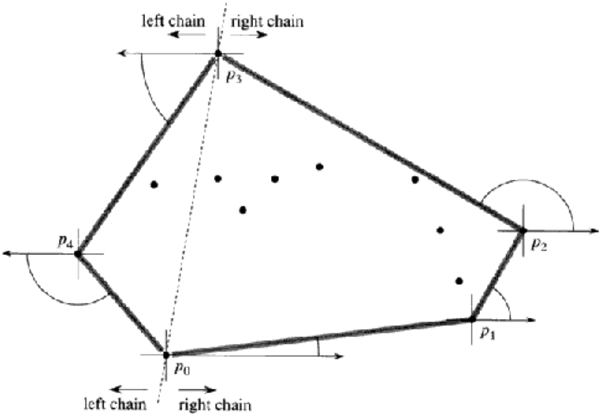

Package Wrapping – Jarvis March

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$$\sum_{t=1}^{\lg \lg h} n2^t = n \sum_{t=1}^{\lg \lg h} 2^t \leq n2^{1+\lg \lg h} = 2n \lg h = O(n \log h)$$