Computational Geometry

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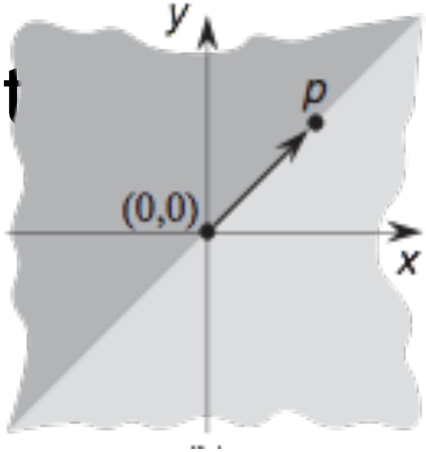
3 important tests

- Given 2 vectors ab and ac, is ab clockwise from ac with respect to a?
- If we traverse from a to b and then to c, do we make a left turn at b?
- Do segments ab and cd intersect



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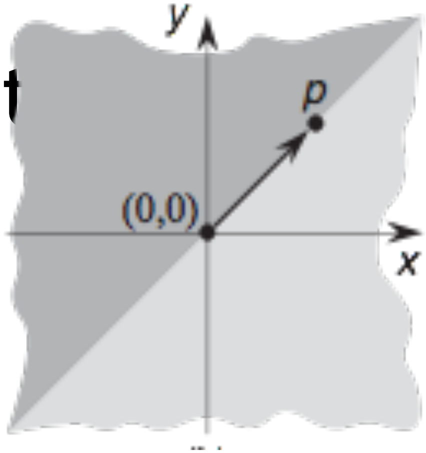
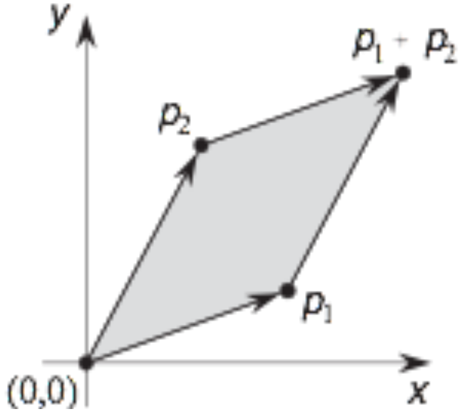


Cross Products



- Let a = origin (0,0)
- Let p1 = vector from a to b
- Let p2 = vector from a to c
- Cross product = signed area of parallelogram
- p1 X p2 has magnitude = |x1 y2 – x2 y1|
- p1 X p2 has direction normal to p1 and p2.
  - Use right hand rule

Cross Products & “Clockwiseness”

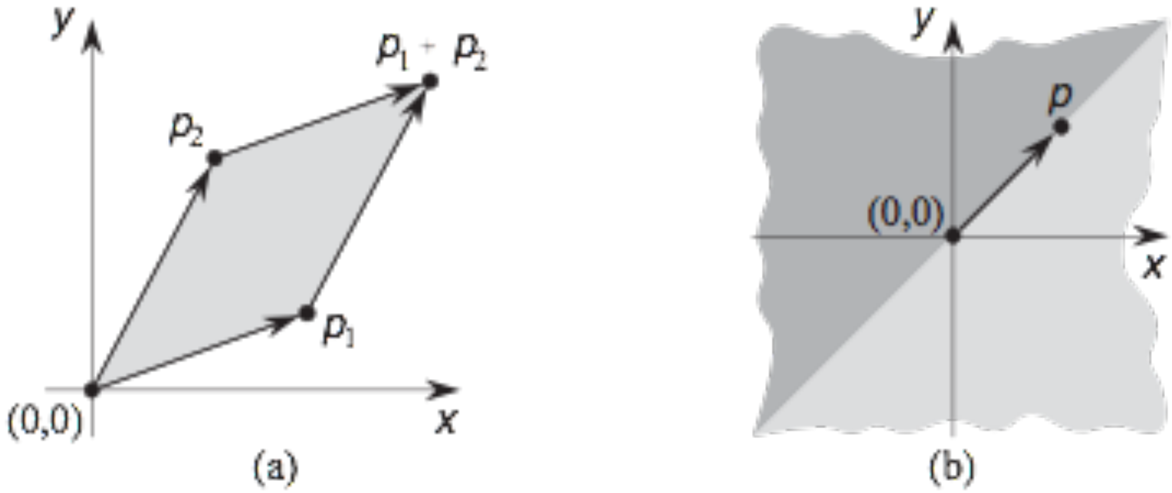
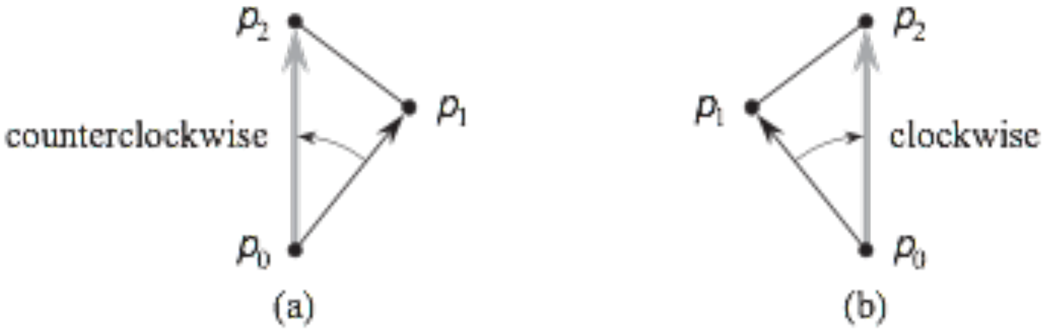


Figure 33.1 **(a)** The cross product of vectors p_1 and p_2 is the signed area of the parallelogram. **(b)** The lightly shaded region contains vectors that are clockwise from p. The darkly shaded region contains vectors that are counterclockwise from p.

“Clockwiseness”



$.p_1 \quad p_0/ \quad .p_2 \quad p_0/ \; D \; .x_1 \quad x_0/.y_2 \quad y_0/ \quad .x_2 \quad x_0/.y_1 \quad y_0/ \; :$

If this cross product is positive, then p_0p_1 is clockwise from p_0p_2; if negative, it is counterclockwise.

```
DIRECTION.$p_i$ ; $p_j$ ; $p_k$/
return .$p_k$   $p_i$/   .$p_j$   $p_i$/
```

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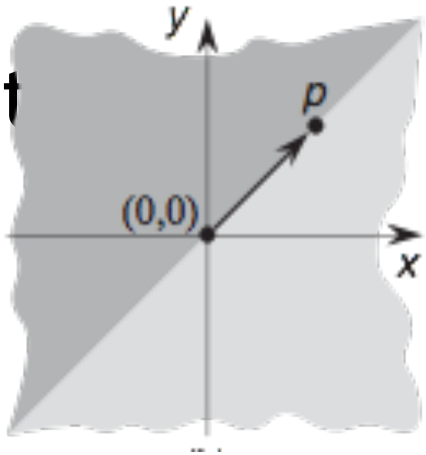
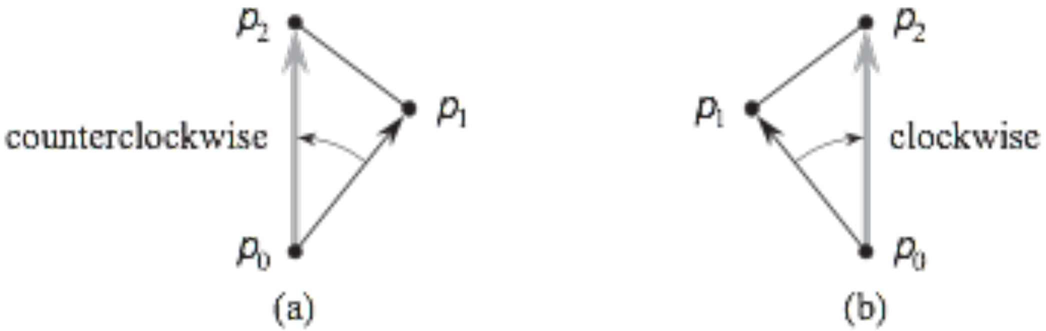


Left-turn test using “clockwiseness”



$.p_1 \quad p_0/ \quad .p_2 \quad p_0/ \; D \; .x_1 \quad x_0/.y_2 \quad y_0/ \quad .x_2 \quad x_0/.y_1 \quad y_0/ \; :$

If this cross product is positive, then p_0p_1 is clockwise from p_0p_2; if negative, it is counterclockwise.

```
DIRECTION.$p_i$ ; $p_j$ ; $p_k$/
    return .$p_k$   $p_i$/   .$p_j$   $p_i$/
```

If DIRECTION(p_i, p_j, p_k) is positive, then LEFT-TURN(p_i, p_j, p_k) is true

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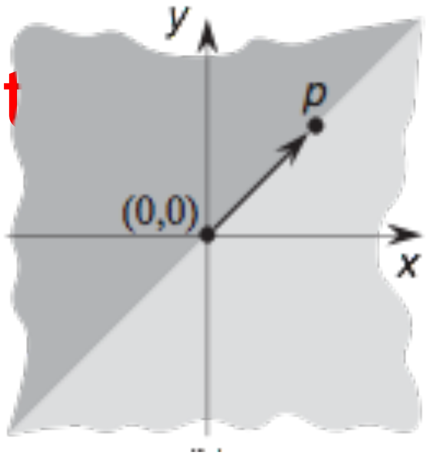
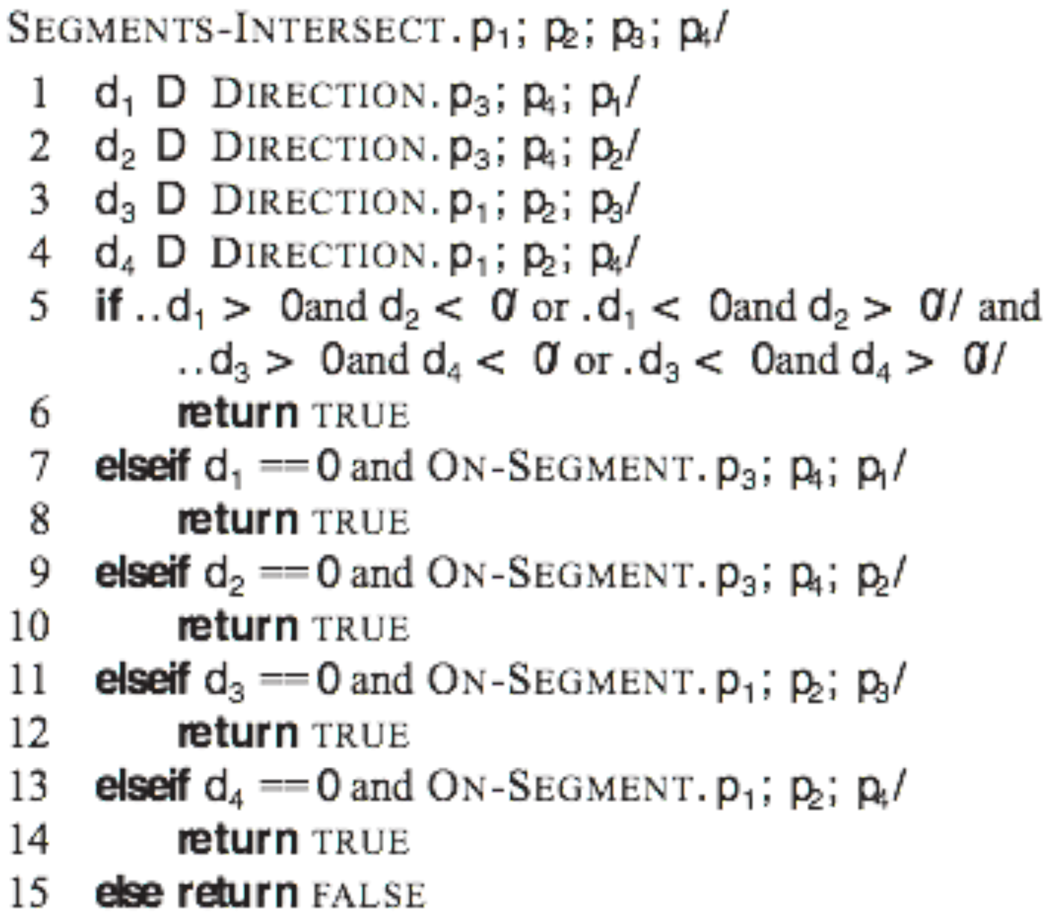


Segment Intersection Test

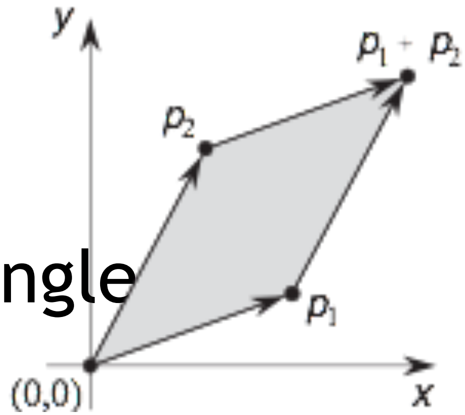
- Standard method
  - Write down equations of two lines
  - Find intersection point
  - If one is found, then the segments intersect
  - Else, they don’t intersect
- How can we solve segment intersection using the LEFT-TURN test?

Segment Intersection



```
SEGMENTS-INTERSECT.$p_1$; $p_2$; $p_3$; $p_4$/
$d_1$ D DIRECTION.$p_3$; $p_4$; $p_1$/
$d_2$ D DIRECTION.$p_3$; $p_4$; $p_2$/
$d_3$ D DIRECTION.$p_1$; $p_2$; $p_3$/
$d_4$ D DIRECTION.$p_1$; $p_2$; $p_4$/
if ..$d_1$ > 0and $d_2$ < 0/ or .$d_1$ < 0and $d_2$ > 0// and
        ..$d_3$ > 0and $d_4$ < 0/ or .$d_3$ < 0and $d_4$ > 0//
        return TRUE
elseif $d_1$ == 0 and ON-SEGMENT.$p_3$; $p_4$; $p_1$/
        return TRUE
elseif $d_2$ == 0 and ON-SEGMENT.$p_3$; $p_4$; $p_2$/
        return TRUE
elseif $d_3$ == 0 and ON-SEGMENT.$p_1$; $p_2$; $p_3$/
        return TRUE
elseif $d_4$ == 0 and ON-SEGMENT.$p_1$; $p_2$; $p_4$/
        return TRUE
else return FALSE
```

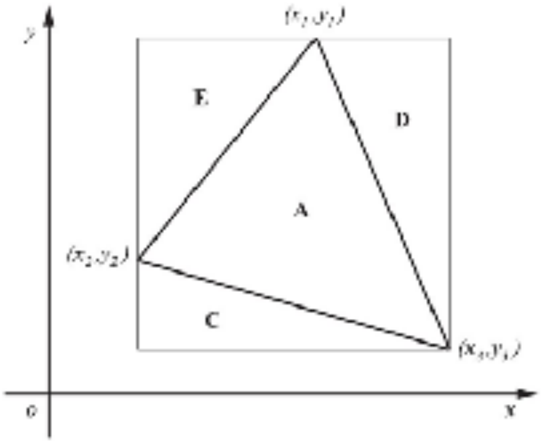
Area of a Triangle



- Area = Base X Height / 2
- Area = a X b X sin(C) / 2
  - a, b are side lengths, C is internal angle
- Area = sqrt{s (s-a) (s-b) (s-c)},
  - a,b,c are side lengths and s = half of perimeter
- Area = ½ (cross product magnitude)
  - Area = ½ |x1 y2 – x2 y1|
  - Assumes one vertex is the origin

Area of a Triangle

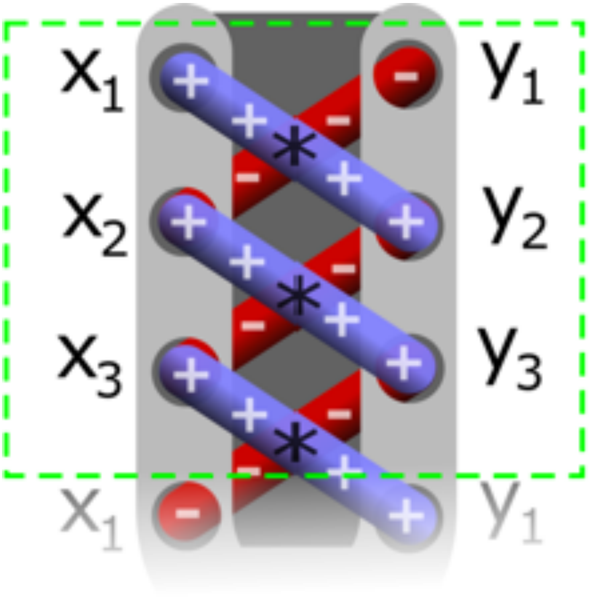
- Area = R – C – D – E
- $\mathbf{R} = (x_3 - x_2)(y_1 - y_3) = (x_3 y_1 + x_2 y_3) - (x_3 y_3 + x_2 y_1)$
- $\mathbf{A} = \frac{1}{2}((x_2 y_3 - x_3 y_2) - (x_1 y_3 - x_3 y_1) + (x_1 y_2 - x_2 y_1))$
- $$\mathbf{A} = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$
- $\mathbf{A} = \frac{1}{2}|x_1 y_2 + x_2 y_3 + x_3 y_1 - x_2 y_1 - x_3 y_2 - x_1 y_3|$



Area of Polygon: Shoelace Formula

$$\mathbf{A} = \frac{1}{2}\left|\sum_{i=1}^{n-1} x_i y_{i+1} + x_n y_1 - \sum_{i=1}^{n-1} x_{i+1} y_i - x_1 y_n\right|$$

$$= \frac{1}{2}\left|x_1 y_2 + x_2 y_3 + \cdots + x_{n-1} y_n + x_n y_1 - x_2 y_1 - x_3 y_2 - \cdots - x_n y_{n-1} - x_1 y_n\right|$$



https://en.wikipedia.org/wiki/Shoelace_formula

Sorting points by polar angle

```
struct Point {int x,y;}

int operator^(Point p1, Point p2) {return p1.x*p2.y - p1.y*p2.x;}

bool operator<(Point p1, Point p2)
{
    if (p1.y == 0 && p1.x > 0) return true; //angle of p1 is 0, thus p2>p1
    if (p2.y == 0 && p2.x > 0) return false; //angle of p2 is 0 , thus
p1>p2
    if (p1.y > 0 && p2.y < 0) return true; //p1 is in [0..180], p2 in
[180..360]
    if (p1.y < 0 && p2.y > 0) return false;
    return (p1^p2) > 0; //return true if p1 is clockwise from p2
}
```