Convex Hull

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Convex Regions

• **Convex region**: A region in space is called **convex** if line joining any two points in the region is completely contained in the region.
Non-convex polygons

- Convex vs Non-convex
Convex Hulls and Polygons

- **Convex hull** of a set of points, $S$, is the smallest convex region containing $S$. 
Rubber Band Analogy for Convex Hulls
Tangents to Polygons

Tangents from a point

Tangents from a polygon
Graham Scan
Convex Hull: Graham Scan applet

- http://www.personal.kent.edu/~rmuhamma/Compgeometry/MyCG/ConvexHull/GrahamScan/grahamScan.htm
  - Main cost: sorting
    - O(n log n)
Package Wrapping: Jarvis March
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- **Time complexity**
  - (Cost of iteration) $\times$ (# iterations)
- **Each iteration:** $O(n)$
- **Number of iterations =** $O(n)$
- **Cost =** $O(nh)$
  - $h$ = # of points on convex hull
Complexity of Convex Hull

- Graham Scan: $O(n \log n)$
- Jarvis March: $O(nh)$  [output sensitive]
- Lower Bound = $\Omega(n \log h)$
Other Methods

- Divide and Conquer
- Conquer and Divide
- Randomized algorithms
Chan’s Algorithm

- Combines the benefits of both algorithms
- Partition points into $n/m$ groups of size $m$
- Use Graham scan on each one
  - $O((m \log m) (n/m)) = O(n \log m)$
- Merge the $n/m$ convex hulls using a Jarvis march algorithm by treating each group as a “big point”
  - Tangent between a point and a convex polygon with $m$ points can be computed in $O(\log m)$ time
  - $O((n/m)(\log m)(h)) = O((n/m)h \log m)$
Chan’s Algorithm

- Time Complexity = $O(n \log m + (n/m) h \log m)$
- If $m = h$, then time = $O(n \log h)$
- How to guess $h$?
  - Linear Search
    - Time complexity = $O(nh \log h)$
  - Binary Search
    - Time complexity = $O(n \log^2 h)$
  - Doubling Search ($m = 1, 2, 4, 8, ...$)
    - Time Complexity = $O(n \log^2 h)$
  - ???
Chan’s Algorithm: More tricks

• What if $m = h^2$?
  - Then $O(n \log m) = O(n \log h)$

• So try: $m = 2, 4, 16, 256, ...$

\[
\sum_{t=1}^{\lg \lg h} n2^t = n \sum_{t=1}^{\lg \lg h} 2^t \leq n2^{1+\lg \lg h} = 2n \lg h = O(n \log h),
\]
3D convex hulls