Two Stones
kattis: twostones

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Game of “Two Stones”

- Given N stones arranged sequentially, Alice and Bob play a game as follows
  - In each turn they pick exactly 2 adjacent stones
  - If no more pairs of adjacent stones exist, game ends
  - **Alice wins** if
    - number of stones remaining is odd
  - Else **Bob wins**
- Output the winner “Alice” or “Bob” without quotes on a line
Who wins with ... 

- $N = 1$
- $N = 2$
- $N = 3$
- $N = 4$
- $N = \text{odd}$
- $N = \text{even}$
Challenges

• N has a large range: N <= 10 million
• There are 2 possible starts depending on who goes first
• For large N, there are many, many moves for each player at each step
• Cannot simulate every possible game to figure out who the winner would be
Method of Reduction

• For each possible scenario, what happens when one of them moves?
  ▪ How does the situation change?
  ▪ How does it affect who will win?
  ▪ What is a good move? Bad move?
Invariant

- Alice wins if the number of lone stones are odd in number
- Alice wins if there are an odd number of odd sequences
- Regardless of who plays, the parity of the number of odd sequences remains the same
  - If it started with even, it stays even. **HOW?**
  - If it started with odd, it stays odd
Final Solution

• We only need to count the number of odd sequences in the initial set
• Since there is only one sequence of length $N$ at start, the answer is simple
  - If $N = \text{odd}$, Alice wins
  - Else, Bob wins
What if …

• What if Alice’s reward is equal to the number of stones left over?
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