Coding Theory Framework for Target Location in Distributed Sensor Networks[1]

Krishnendu Chakrabarty
Dept. Electrical & Computer Engineering
Duke University
Durham, NC 27708

Sitharama S. Iyengar
Dept. Computer Science
Louisana State University
Baton Rouge, LA 70803

Hairong Qi
Dept. Electrical & Computer Engineering
University of Tennessee
Knoxville TN 37996

Eungchun Cho
Dept. Mathematics and Sciences
Kentucky State University
Frankfort, KY 40601

Abstract

Distributed, real-time sensor networks are essential for effective surveillance in the digitized battlefield and for environmental monitoring. In this paper, we present the first systematic theory that leads to novel sensor deployment strategies for effective surveillance and target location. We represent the sensor field as a grid (two- or three-dimensional) of points (coordinates), and use the term target location to refer to the problem of pin-pointing a target at a grid point at any instant in time. We use the framework of identifying codes to determine sensor placement for unique target location. We provide coding-theoretic bounds on the number of sensors and present methods for determtheir placement in the sensor field. We also show that sensor placement for single targets provides asymptotically complete (unambiguous) location of multiple targets.

1 Introduction

Distributed, real-time sensor networks are essential for effective surveillance in the digitized battlefield and for environmental monitoring. An important problem in sensor networks is that of target location. If the sensor field is represented as a grid (two- or three-dimensional) of points (coordinates), target location refers to the problem of pin-pointing a target at a grid point at any point in time. For enhanced coverage, a large number of sensors are typically deployed in the sensor field, and if the coverage areas of multiple sensors overlap, they may all report a target in their respective zones. The precise location of the target must then be determined by examining the location of these sensors. In many cases, it is even impossible to precisely locate the target (within the granularity of a single grid point). Alternatively, target location can be simplified considerably if the sensors are placed in such a way that every grid point in the sensor field is covered by a unique subset of sensors. In this way, the set of sensors reporting a target at time t uniquely identify the grid location for the target at time t. The trajectory of a moving target can also be easily determined in this fashion from time series data.

Previous research in distributed sensor networking has largely ignored the above sensor placement issues. Most prior work has concentrated exclusively on efficient sensor communication [1, 2] and sensor fusion [3, 4, 5] for a given sensor field architecture. However, as sensors are used in greater numbers for field operation, efficient deployment strategies become increasingly important. Indeed, it is fair to state that the extensive research in this area has not yet led to a firm grasp of sensor deployment strategies for target location. This lack of understanding is not altogether surprising because the sensor deployment combines the hitherto unexplained interaction of target location with optimal placement of sensors.

The sensor placement problem for target location is closely related to the alarm placement problem described in [6]. The latter refers to the problem of placing "alarms" on the nodes of a graph G such that a single faults in the system (corresponding to a single faulty node in G) can be diagnosed. The alarms are therefore analogous to sensors in a sensor field. It was shown in [6] that the alarm placement problem is NP-complete for arbitrary graphs. However, we show that for restricted topologies, e.g. a set of grid points in a sensor field, a coding theory framework can be used to efficiently determine sensor placement. The sensor locations correspond to codewords of an identifying code constructed over the grid points in the sensor field. Such coding

[1]This research was supported in part by DARPA under contract no. N66001-001-8946.

frameworks are often used in computing systems, e.g. for error control [7] and more recently for resource placement in multicomputers [8].

In this paper, we address the following sensor deployment problem: How should the sensors be placed at grid points such that every grid point is covered by a unique subset of sensors. We use the theoretical framework of identifying codes [9] to determine the best placement of sensors such that the grid point for a target can be uniquely identified. To the best of our knowledge, this paper presents the first systematic theoretical formulation and practical solutions to these sensor deployment problems.

2 Sensor placement for target location

In this section, we address the problem of placing sensors on grid points such that the grid positions of targets can be uniquely identified from the subset of sensors that detect the targets. This approach is based on the concept of identifying codes for uniquely identifying vertices in graphs [9].

The identifying code problem can be stated as an optimal covering of vertices in an undirected graph G such that any vertex in G can be uniquely identified by examining the vertices that cover it. A *ball* of radius r centered on a vertex v is defined as the set of vertices that are at distance at most r from v. The vertex v is then said to cover itself and every other vertex in the ball with center v. The formal problem statement is as follows: Given an undirected graph G and an integer $r \geq 1$, find a (minimal) set C of vertices such that every vertex in G belongs to a unique set of balls of radius r centered at the vertices in C. The set of vertices thus obtained constitutes a code for vertex identification.

We now show that the problem of placing sensors for unique target identification can be solved using the theory of identifying codes. The grid points in the sensor field correspond to the vertices in the graph G, while the centers of the balls correspond to the grid points where sensors are placed. The unique identification of a vertex in G corresponds to the unique location of a target by the sensors in the sensor field. Each sensor at a grid point can detect a target at grid points that are adjacent to it.

Let S_n^p denote the number of sensors required for uniquely identifying targets in an n-dimensional ($n \leq 3$) sensor field with p grid points in each dimension. The following theorem provides upper and lower bounds on S_n^p. Its proof follows from the properties of identifying codes on regular graphs [9].

Theorem 1 *The number of sensors S_n^p for uniquely identifying a target in an n-dimensional sensor field with p grid points in each dimension is given by:*

$$\frac{p^n}{n+1} \leq S_n^p \leq \frac{p^n}{n} \qquad (1)$$

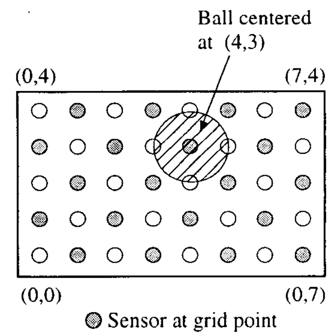


Figure 1. A checkerboard placement of sensors.

For example, for a two-dimensional sensor field with 100 grid points in each dimension, at least 3,334 sensors are required for the 10^4 grid points. However, 5,000 sensors are adequate for unique target identification. For a two-dimensional sensor field, the upper bound corresponds to a checkerboard placement of sensors on grid points as shown in Figure 2. The grid points are marked by their (x, y) coordinates, and each sensor can detect a target at distances upto the next grid point in each dimension. Note that each grid point for this placement is covered by a unique subset of sensors.

We now describe more efficient sensor placement strategies based on coding theory principles from [9]. We first review some terminology. For every grid point (x, y, z) in a sensor field, we associate a parity vector (p_x, p_y, p_y) given as follows:

$$\begin{aligned} p_x &= x \bmod 2 \\ p_y &= y \bmod 2 \\ p_z &= z \bmod 2 \end{aligned}$$

For example, the parity vector for grid point (2,4,5) in a three-dimensional sensor field is (0,0,1). The set of parity vectors is called the binary parity code and denoted by $\mathcal{P}(\mathcal{C})$.

Theorem 2 *For an 3-dimensional sensor field with p grid points in each dimension, p even and $p > 2$, target location is achieved with a smallest possible number of sensors ($S_n^p = p^n/4$) if the binary parity code $\mathcal{P}(\mathcal{C})$ is the perfect binary $(3, 1, 3)$ Hamming code, where a perfect (n, k, d) Hamming code consists of 2^k codewords in n dimensions and the minimum distance between codewords is d.*

Proof: We first prove that every grid point is uniquely covered. Every sensor is covered only by itself because the Hamming distance between any two parity vectors is at least three. Next consider a noncodeword vertex with coordinates (x_1, x_2, x_3) and corresponding parity vector (p_1, p_2, p_3). There are two vertices with coordinates $x' = (x_1', x_2', x_3')$

and $x'' = (x''_1, x''_2, x''_3)$ such that have the same parity vector (q_1, q_2, q_3), x' and x'' are neighbors of x in the n-dimensional sensor field, (q_1, q_2, q_3) belongs to the Hamming code, and the Hamming distance between (p_1, p_2, p_3) and (q_1, q_2, q_3) is one. We note that x' and x'' are uniquely determined by x.

To prove necessity, we note that if two sensors in the p-ary n-dimensional sensor field are neighbors, their parity vectors are at distance 1. Thus, for an identifying code, the covering radius of the set of parity vectors must be equal to 1, and the smallest set with this property is a perfect (3, 1, 3) code. □

The following theorem shows that if the number of grid points in each dimension is even, the lower bound on the number of sensors (Theorem 1) can be achieved for a three-dimension sensor field. The proof follows from Theorem 2.

Theorem 3 *For a three-dimensional sensor field with p grid points ($p > 4$, p even) in each dimension, sensor placement with a minimum number of sensors ($S_n^p = p^3/4$) can be achieved if and only if sensors are placed on grid points whose parity vectors are (0,0,0) and (1,1,1).*

Theorem 2 shows that if p is even, the sensor density (average number of sensors per grid point) for three-dimensional sensor fields is only 0.25. For example, let $p = 6$. From Theorem 2, we see that sensors should be placed at the set of grid points $\{S_0, S_1\}$, where S_0 and S_1 are the set of grid points with parity vectors (0,0,0) and (1,1,1), respectively, as shown below:

S_0 ={(0,0,0), (0,0,2), (0,2,0), (0,2,2), (0,0,4), (0,4,0), (0,4,2), (0,2,4), (0,4,4), (2,0,0), (2,0,2), (2,2,0), (2,2,2), (2,0,4), (2,4,0), (2,2,4), (2,4,4), (4,0,0), (4,0,2), (4,2,0), (4,2,2), (4,0,4), (4,4,0), (4,4,2), (4,2,4), (4,4,4)}

S_1 ={(1,1,1), (1,3,1), (1,3,1), (1,3,3), (1,1,5), (1,5,1), (1,5,3), (1,3,5), (1,5,5), (3,1,1), (3,1,3), (3,3,1), (3,3,3), (3,1,5), (3,5,1), (3,3,5), (3,5,5), (5,1,1), (5,1,3), (5,3,1), (5,3,3), (5,1,5), (5,5,1), (5,5,3), (5,3,5), (5,5,5)}.

Hence, a total of 54 sensors are required for the 216 grid points.

The next theorem addresses cases where p is not necessarily even. For a sensor field with p grid points in each dimension, we can define an n-dimensional p-ary code C with covering radius 2 as follows: C is the smallest set of grid points (vertices) such that each non-codeword is at distance at most two from a codeword. Note that the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in this context is given by $d = |x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2|$.

Theorem 4 *Let $K^p(n, 2)$ be the minimum number of codewords in a p-ary n-dimensional code with covering radius 2. Then for any $p > 4$, an upper bound on the minimum number of sensors S_n^p for target location in an n-dimensional sensor field with p grid points is given by*

$$S_n^p \leq (2n + 1)K^p(n, 2) \qquad (2)$$

To prove this theorem, it is sufficient to show that all grid points in a ball B_2 of radius 2 with center v can be uniquely identified by balls of radius 1 centered at all gridpoints that belong to the ball B_1 of radius 1 centered at v. Without loss of generality, we can assume that $v = (0, 0, \ldots, 0)$. Then $B_1 = \{(0, 0, \ldots, 0)\} \bigcup \{(0, \ldots, 0, \pm 1, 0, \ldots, 0) \pmod p\}$ and $B_2 = B_1 \bigcup \{(0, \ldots, 0, \pm 2, 0, \ldots, 0) \pmod p\} \bigcup \{(0, \ldots, \pm 1, 0, \ldots, 0, \pm 1, 0, \ldots, 0) \pmod p\}$. Let $x \in B_2$. We have to consider the following four cases:

1) $x = (0, \ldots, 0)$. Then x belongs to all balls of radius 1 with centers in B_1.

2) $x = (0, \ldots, 0, \pm 1, 0, \ldots, 0)$. Then x belongs to two balls of radius 1 with centers at x and $(0, \ldots, 0)$, respectively.

3) $x = (0, \ldots, 0, \underbrace{\pm 1}_{i}, 0, \ldots, \underbrace{\pm 1}_{j}, 0, \ldots, 0)$. Then x belongs to 2 balls with centers $(0, \ldots, 0, \underbrace{\pm 1}_{i}, 0, \ldots, 0)$ and $(0, \ldots, 0, \underbrace{\pm 1}_{j}, 0, \ldots, 0)$.

4) $x = (0, \ldots, 0, \underbrace{\pm 2}_{i}, 0, \ldots, 0)$. Then x belongs to one ball with center $(0, \ldots, 0, \underbrace{\pm 1}_{i}, 0, \ldots, 0)$.

This completes the proof. □

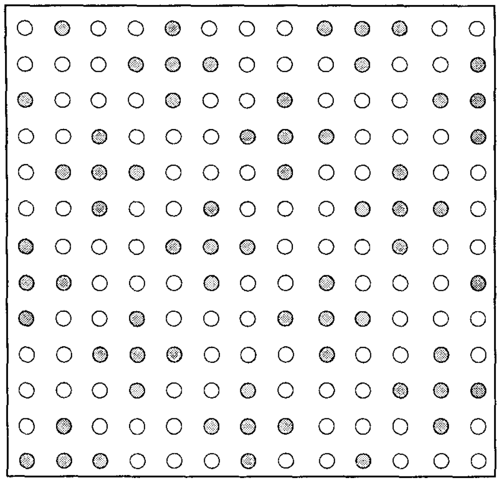
Theorem 4 implies that sensor placement can be carried out by first determining a code $K^p(n, 2)$ with covering radius 2. (Tables of covering codes are easily available [10].). Sensors are then placed on the grid points corresponding to the codewords as well as on all grid points that are adjacent to codewords of $K^p(n, 2)$. This is shown in Figure 1 for a two-dimensional sensor field with $p = 13$. We need a total of 65 sensors for 169 grid points (sensor density = 0.38), which is slightly greater than the lower bound of 57 predicted by Theorem 1. Note however that the lower bound need not always be achievable.

As an another example, let $p = 5$ and $n = 3$. For this case, $K^5(3, 2) = 5$ [10, 11], hence a total of 35 sensors placed at the 125 grid points provides unique target location.

While the above sensor deployment strategy can be used in general for any $p > 4$, the sensor density can often be decreased for specific values of p. For example, consider the special case $p = 8s$ and $n = 2$. An ad hoc sensor placement given by Figure 3 yields a sensor density of only 0.375, which improves upon the construction of Theorem 4.

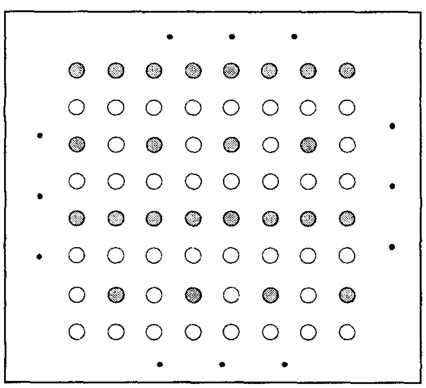
3 Locating multiple targets

We have assumed thus far that the location of only a single target in the sensor field has to be uniquely identified. We now show that sensor placement for unique location of single target provides a near-complete location of sets of targets. This demonstrates that the sensor placement strategy outlined in this section is effective even for tracking



Sensor at grid point

Figure 2. An efficient placement of sensors given by Theorem 3.



Sensor at grid point

Figure 3. An efficient ad hoc placement of sensors.

multiple targets in the sensor field. Let $C(l)$ be the fraction of sets of targets of cardinality exactly l that are uniquely identifiable. The following lemma provides a lower bound on the fraction of multiple targets that can be located.

Lemma 1 *The fraction $C(l)$ of sets of targets of cardinality exactly l that are uniquely identifiable with $t = 1$ by sensor placement for single targets is lower-bounded by $C(l) \geq \Pi_{i=0}^{l-1} \frac{N - iV(4)}{N - i}$, where $V(4)$ is the number of grid points at distance upto four grid points from any given vertex in the graph, and N is the number of grid points in the sensor field.*

Proof: A set of targets is uniquely identifiable if the distance between any two targets (grid points) in this set is at least five. Note that this condition is sufficient but not necessary. The fraction of identifiable sets of vertices is therefore lower-bounded by

$$C(l) \geq \frac{N(N - V(4))(N - 2V(4)) \cdots (N - (l-1)V(4))}{\binom{N}{l} l!}$$

$$= \Pi_{i=0}^{l-1} \frac{N - iV(4)}{N - i}.$$

This completes the proof of the lemma. □

The above lemma can be used to show that if the number of grid points is sufficiently large relative to the cardinality of the set of targets, the multiple targets can be uniquely located (asymptotically) using sensor placement for single targets.

Theorem 5 *As the number of grid points in a sensor field tends to infinity, the fraction of sets of targets of cardinality exactly l that are uniquely identifiable approaches one if $l = o(\sqrt{N})$.*

Proof: Let $\prod = \Pi_{i=0}^{l-1}(\frac{N - iV(4)}{N - i})$. It can be easily seen that for $i \lesssim \sqrt{N}$,

$$\ln \frac{N - iV(4)}{N - i} = \ln(1 - \frac{i(V(4) - 1)}{N - i}) \sim -\frac{i(V(4) - 1)}{N - i},$$

and $\ln \prod \sim \sum_{i=1}^{l-1} -\frac{i(V(4) - 1)}{N - i}$. Now,

$$\left|\sum_{i=1}^{l-1} \frac{i(V(4) - 1)}{N - i}\right| \leq \frac{(l-1)(V(4) - 1)}{N - l + 1}(l - 1),$$

and $\lim_{N \to \infty} \frac{(l-1)(V(4) - 1)}{N - l + 1}(l - 1) = 0$ if $l^2/N \to 0$ (since $V(4)$ is constant). □

This underlines the effectiveness of the sensor placement approach for single targets, and implies that separate placement algorithms for multiple targets are not necessary.

4 Conclusions

We have presented novel sensor deployment strategies for effective surveillance and target location. This approach represents the sensor field as a grid (two- or three-dimensional) of points (coordinates), and sensors are selectively placed on a subset of these grid points. We have used the framework of identifying codes to determine sensor placement for unique target location, which refers to the problem of pin-pointing a target at a grid point at any instant in time. We have provided coding-theoretic bounds on the number of sensors and presented methods for determining their placement in the sensor field. We have also shown that sensor placement for single targets provide asymptotically complete (unambiguous) location of multiple targets.

References

[1] J. M. Kahn, R. H. Katz and K. S. J. Pister. Mobile networking for smart dust. *ACM/IEEE International Conference on Mobile Computing and Networks*, 1999.

[2] D. Estrin, R. Govindan, J. Heidemann, S. Kumar. Next century challenges: Scalable coordination in sensor networks *Proc. ACM/IEEE International Conference on Mobile Computing and Networks*, 1999.

[3] R. R. Brooks and S. S. Iyengar. *Multi-Sensor Fusion: Fundamentals and Applications with Software*, Prentice Hall PTR, Upper Saddle River, NJ, 1998.

[4] S. S. Iyengar, L. Prasad and H. Min. *Advances in Distributed Sensor Technology*, Prentice-Hall, Englewood Cliffs, NJ, 1995.

[5] H. Qi, S. S. Iyengar and K. Chakrabarty. Distributed multi-resolution data integration using mobile agents, accepted for publication in *Proc. IEEE Aerospace Conference*, 2001.

[6] N. S. V. Rao. Computational complexity issues in operative diagnosis of graph-based systems. *IEEE Transactions on Computers*, vol. 42, pp. 447–457, April 1993.

[7] T. R. N. Rao. *Error Control Coding for Computer Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1989.

[8] M. Bae and B. Bose. Resource placement in torus-based networks. graph-based systems. *IEEE Transactions on Computers*, vol. 46, pp. 1083–1092, October 1997.

[9] M. G. Karpovksy, K. Chakrabarty and L. B. Levitin. *IEEE Transactions on Information Theory*, vol. 44, pp. 599–611, March 1998.

[10] G. D. Cohen, I. Honkala, S. N. Litsyn and A. Lobstein. *Covering Codes*, North Holland, Amsterdam, The Netherlands, 1997.

[11] P. R. J. Östergård. Upper bounds on q-ary covering codes. *IEEE Transactions on Information Theory*, vol. 37, pp. 660–664, May 1991.