This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2010 proceedings.

A Novel Robust Detection Algorithm Using Jarqur-Bera Statistic for Spectrum Sensing

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***Abstract*—In this paper, the DTV (digital television) spectrum sensing problem is studied, which plays a key role in the cognitive radio. In contrast to the existing higher-order-statistics (HOS) approach, we propose a novel robust spectrum-sensing method, which is based on the JB (Jarqur-Bera) statistic. In our new studies, the existing HOS spectrum sensing technique may often not be robust. Our proposed JB-statistic based detector has been justified to be superior for the simulated microphone signals as well as the real DTV signals.**

***Index Terms*—Spectrum sensing, signal detection, higher-order-statistics (HOS), JB (Jarqur-Bera) statistic, DTV.**

I. Introduction

The increasing demand for wireless connectivity and the crowded unlicensed spectra have prompted the regulatory agencies to be more aggressive in coming up with new ways to use spectra more wisely [1]. Hence, cognitive radio arises as a feasible solution to the aforementioned spectral congestion problem by introducing the opportunistic usage of the frequency bands that are not heavily occupied by licensed users [2].

As a consequence, a working group named IEEE 802.22 was established to develop a standard for a cognitive radio based PHY/MAC/air interface to target on the license-exempt devices. The IEEE 802.22 WRAN Group established a sensing tiger team in charge of the spectrum sensing for the DTV signals, which are modulated according to the ATSC digital television standard [3]. In addition, wireless microphones are the usual low-power secondary licensed signals operating in the locally unused DTV bands. Therefore, the main task in spectrum sensing for the IEEE 802.22 WRAN systems is to detect the existence of the DTV signals as well as the wireless microphone signals operating in the DTV frequency bands.

There exist several spectrum sensing techniques, including the energy detection approach [4], the matched filtering approach [1], and the cyclostationary detection approach [5]. Each approach should rely on different operational requirements. They all have advantages and disadvantages. For example, the cyclostationary detection approach requires the knowledge of the cyclic frequencies associated with the primary users, while the matched filtering approach needs to know the waveforms and the channel impulse responses of the primary users. On the other hand, the energy detection approach does not need any information regarding the signal to be detected and hence it is robust to the unknown dispersive channel. However, the energy detection approach is much vulnerable to the noise uncertainty [4]. To overcome the shortcomings of the energy detection approach, some methods based on the eigenvalues associated with the covariance matrix of the received signal were proposed in [2], [6]. However, the corresponding computational complexities are quite large. A method based on the higher-order-statistics (HOS) was proposed and it would be promising especially in the low SNR conditions [7]. In this paper, we propose a novel spectrum sensing scheme which is based on the Jarque-Bera (JB) statistic. Our method greatly outperforms the HOS technique over all SNR conditions. Furthermore, our method can perform very well even when the SNR is very low and the signal sample size is not large based on the Monte Carlo simulations. Our proposed new spectrum sensing technique would have a great potential to serve as the backbone of the future cognitive radio technology.

This work was supported by Information Technology Research Award for National Priorities (NSF-ECCS 0426644) from National Science Foundation, Faculty Research Initiation Award from the Southeastern Center for Electrical Engineering Education, Research Enhancement Award from the Louisiana-NASA Space Consortium and Faculty Research Award from Louisiana State University.

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II. System Model

Denote the continuous-time received signal by $r_c(t)$ during the sensing stage. The underlying signal from the primary users is denoted by $s_c(t)$ and $w_c(t)$ is the additive white Gaussian noise (AWGN). Hence, we have

$$r_c(t) \overset{\text{def}}{=} s_c(t) + w_c(t). \tag{1}$$

Assume that we are interested in the frequency band with the central frequency f_c and the bandwidth W. We sample the received signal at a sampling rate f_s, where $f_s \geq W$. Let $T_s = \frac{1}{f_s}$ be the sampling period and N be the sample size. For notational convenience, we denote

$$\begin{aligned} r(n) &\overset{\text{def}}{=} r_c(nT_s), \;\; n = 1, \ldots, N, \\ s(n) &\overset{\text{def}}{=} s_c(nT_s), \;\; n = 1, \ldots, N, \\ w(n) &\overset{\text{def}}{=} w_c(nT_s), \;\; n = 1, \ldots, N. \end{aligned} \tag{2}$$

For the signal detection (spectrum sensing) problem, there involve two hypotheses, namely H_0: signal is absent and H_1: signal is present. The discrete-time received signals under these two hypotheses are given by

$$H_0 : r(n) = w(n), \tag{3}$$

$$H_1 : r(n) = s(n) + w(n), \tag{4}$$

where $r(n)$ denotes the received signal samples including the effect of path loss, multipath fading and time dispersion, and

$w(n)$ is the discrete-time AWGN with zero mean and variance σ^2. Here $s(n)$ can be the superposition of the signals emitted from multiple primary users. It is assumed that signal and noise are uncorrelated with each other. The spectrum sensing (or signal detection) problem is therefore to determine whether the signal $s(n)$ exists or not, based on the received signal samples $r(n)$ [2], [6]. In reality, the recorded DTV channels are sampled at $f_s = 21.524476$ MHz and then down-converted to a low central intermediate frequency (IF) of 5.381119 MHz (one fourth of the sampling frequency) [8]. The acquired signal samples are used to detect if any DTV signal exists.

III. Efficient Spectrum Sensing Techniques

The computationally-efficient signal detection methods using the partial statistics have been attracting a lot of research interest for decades. In this section, we first present an existing spectrum sensing technique based on the higher-order statistics. Then, we propose a novel spectrum sensing algorithm based on the JB-statistic, which is more robust than the former method especially when the sample size of the received signal is quite small.

A. Higher-Order-Statistics Spectrum-Sensing Algorithm

In this subsection, we will discuss about the higher-order-statistics (HOS) based detection algorithm (see [7]). This sensing technique is based on Gaussian noise statistics. The higher-order statistics can be used to evaluate how well the distribution of the test statistic matches a Gaussian distribution. In this method, the received signal is converted down to the baseband and then filtered. Next, the nominal ATSC pilot frequency is aligned to the DC and the down-converted signal is filtered again by a narrow-band low-pass filter. The resultant signal is transformed to the frequency domain using the fast Fourier transform (FFT). Often, a 2,048-point FFT is recommended, since it is also used in the OFDM modulator/demodulator for the digital video broadcasting systems. Then, the higher-order moments and cumulants (higher than the second-order) for the real and imaginary parts of the signal spectra are calculated. If only noise is present, then the real and imaginary parts of the signal spectra are both Gaussian. The corresponding higher-order cumulants are thus all zero. Hence, in this sensing technique, a *Gaussianity test* is performed using the estimates of the higher-order cumulants. If it fails the Gaussianity test, then the hypothesis that the ATSC pilot signal is present holds true. The HOS detection algorithm has to use the third- to sixth-order cumulants and central moments [7]. The estimation variances of such high-order cumulants are usually quite large especially when the sample size is small [9]. Hence, it is obvious that the HOS approach cannot be robust when we do not have much received signal data or the channel model is time-varying. It motivates us to design a new spectrum sensing method to combat this problem.

B. Jarqur-Bera (JB) Statistic Based Detection Algorithm

Our goal is to design a robust spectrum sensing method involving estimates with less variances and leading to a computationally efficient solution. The JB statistic based on skewness and kurtosis is adopted here because kurtosis and skewness, which are composed by the second, third and fourth central moments only, could lead to more robust estimators than the HOS scheme relying on the higher-order moments or cumulants. Since JB statistic only depends on the second to fourth central moments, it would result in much less estimation variance than the variance of the HOS-based estimator (see [7], [9]) using the second to sixth cumulants and central moments altogether according to the k-statistic and h-statistic theory. In addition, the HOS detection method (see [7]) tests the normality using the real and imaginary parts of the complex received samples subject to the property that all higher-order cumulants of a Gaussian PDF are zero. We propose to adopt the JB statistic to work on the norms of the complex signal samples, and the associated normality test is thus subject to the Rayleigh distribution instead. It is well known that the variance of a Gaussian process (for either its real or imaginary part) is much larger than the variance of the corresponding Rayleigh process (constituting the absolute values of the complex Gaussian random data). Hence, it can be foreseen that our proposed spectrum sensing method based on the signal norms can have the advantage over the method in [7]. Our proposed new spectrum sensing algorithm will be presented subsequently in Sections III-B1- III-B3.

1) Pre-Processing: The pre-processing steps in our proposed algorithm for transforming the received signal $r(n)$ into the frequency domain are the same as the HOS detection method [7]. Nevertheless, in our new detection method, we use the Jarque-Bera statistic of the signal spectrum's absolute values. The block diagram of our proposed new spectrum sensing method is depicted in Figure 1.

The signal flow in Figure 1 is described as follows. When the signal $r(n)$ is received, first we multiply $r(n)$ by $e^{-j2\pi f_c n T_s}$ to down-convert it to the baseband, where f_c is the low central IF frequency of 5.381119 MHz. Then, this baseband signal is sent through a digital image rejection low-pass (LP) filter with bandwidth $BW_r = 8 \times 10^6 \times \frac{2\pi}{f_s}$ radians. The image rejection filter is placed in the receiver so that the image frequencies along with other unwanted signals are filtered out to enhance the signal quality.

Next, the enhanced signal $r_2(n)$ is further multiplied by $e^{-j2\pi f_v n T_s}$, where f_v=2.69 MHz. Then, the resulted signal $r_3(n)$ goes through the operations consisting of a down-sampler following a digital anti-aliasing filter whose bandwidth is given by

$$\mathrm{BW}_a = \frac{N_{\mathrm{FFT}}}{T_{\mathrm{sensing}}} \times \frac{2\pi}{f_s}, \tag{5}$$

where N_{FFT} is the FFT window size, and $T_{\mathrm{sensing}} = \frac{n}{f_s}$ is the sensing time. The down-sampling rate f_d is given by

$$f_d = \text{floor}\left(\frac{2\pi}{BW_a}\right), \tag{6}$$

where the function "floor" is the operation to round $\frac{2\pi}{BW_a}$ to the nearest integer less than or equal to $\frac{2\pi}{BW_a}$. The down-sampled signal $r_5(n)$ is sent to a serial-to-parallel port and then the

N_{FFT}-point FFT will be taken to result in a half-period FFT-sequence $R_{\mathrm{out}}(k)$, $k = 0, 1, \ldots, \frac{N_{\mathrm{FFT}}}{2} - 1$.

2) JB-Statistic Based Detection: In statistics, the Jarque-Bera test is a goodness-of-fit measure of departure from normality, based on the sample kurtosis and the sample skewness. The test is named after Carlos M. Jarque and Anil K. Bera. The test statistic, JB, is defined as

$$\mathrm{JB} \stackrel{\mathrm{def}}{=} \frac{n_s}{6}\left(\mathcal{S}^2 + \frac{(\mathcal{K}-3)^2}{4}\right), \tag{7}$$

where n_s is the number of observations (or degrees of freedom in general); $\mathcal{S}$ is the sample skewness and $\mathcal{K}$ is the sample kurtosis. They are defined as

$$\mathcal{S} \stackrel{\mathrm{def}}{=} \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\hat{\mu}_3}{(\hat{\sigma}^2)^{3/2}} = \frac{\frac{1}{n_s}\sum_{i=1}^{n_s}(x_i - \bar{x})^3}{\left(\frac{1}{n_s}\sum_{i=1}^{n_s}(x_i - \bar{x})^2\right)^{3/2}}, \tag{8}$$

$$\mathcal{K} \stackrel{\mathrm{def}}{=} \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\hat{\mu}_4}{(\hat{\sigma}^2)^2} = \frac{\frac{1}{n_s}\sum_{i=1}^{n_s}(x_i - \bar{x})^4}{\left(\frac{1}{n_s}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2}, \tag{9}$$

where $\hat{\mu}_3$ and $\hat{\mu}_4$ are the estimates of the third and fourth central moments, respectively; x_i, $i = 1, \ldots, n_s$ are the observations; $\bar{x}$ is the sample mean and $\hat{\sigma}^2$ is the estimate of the second central moment or the variance. Therefore, this JB test can be considered as a sort of *portmanteau test*, since the four lowest moments about the origin are used jointly for its calculation.

For our proposed spectrum sensing method, we do not use the JB statistic as its original way, which is to judge if the hypothesis of a normal distribution is true. Here we check the absolute values of $R_{\mathrm{out}}(k)$, $k = 0, 1, \ldots, \frac{N_{\mathrm{FFT}}}{2} - 1$. Then, we invoke Eqs. (7), (8), and (9) to calculate the JB statistic of $|R_{\mathrm{out}}(k)|$ and compare it with the threshold r_s to decide if there exists the signal $s(n)$. If JB $> r_s$, we say that the signal exists; otherwise (JB $\leq r_s$), we say that the signal is absent. We will present the theoretical study about how to select the threshold r_s subsequently.

3) Threshold Analysis for Our Proposed Method: In this subsection, we will discuss about how to select the threshold r_s for the proposed JB-statistic-based detection scheme according to both theoretical and heuristical analyses. Let's review the Rayleigh distribution first, which is closely related to our proposed feature $|R_{\mathrm{out}}(k)|$ under the JB test ($|R_{\mathrm{out}}(k)|$ is Rayleigh distributed when signal is absent). The Rayleigh distribution is composed by random complex numbers whose real and imaginary components (x and y) are both identically independently distributed (i.i.d.) Gaussian. The Rayleigh PDF with respect to $r = \sqrt{x^2 + y^2}$ is given by

$$f(r; \sigma_r) = \frac{r}{\sigma_r^2}\exp\left(\frac{-r^2}{2\sigma_r^2}\right), \tag{10}$$

where $r \in [0, +\infty)$, and σ_r is the mode. For the Rayleigh PDF given by Eq. (10), the skewness $\mathcal{S}_{\mathrm{Rayleigh}}$ and the kurtosis $\mathcal{K}_{\mathrm{Rayleigh}}$ are given as follows:

$$\mathcal{S}_{\mathrm{Rayleigh}} = \frac{2\sqrt{\pi}\,(\pi - 3)}{(4-\pi)^{\frac{3}{2}}} \approx 0.631, \tag{11}$$

$$\mathcal{K}_{\mathrm{Rayleigh}} = -\frac{6\pi^2 - 24\pi + 16}{(4-\pi)^2} + 3 \approx 2.755. \tag{12}$$

When there is no signal, the input of the pre-processor (as presented in Section III-B1) is $r(n) = w(n)$. Then, after the pre-processing of the input signal, if there is no aliasing, the output $R_{\mathrm{out}}(k)$, $k = 0, 1, \ldots, \frac{N_{\mathrm{FFT}}}{2} - 1$ will be a complex Gaussian process whose real and imaginary components are both i.i.d. Gaussian. Thus, $|R_{\mathrm{out}}(k)|$, $k = 0, 1, \ldots, \frac{N_{\mathrm{FFT}}}{2} - 1$ will be Rayleigh-distributed. Substituting Eqs. (11) and (12) into Eq. (7), we can calculate the theoretical JB statistic value for Rayleigh distribution as $0.0344 N_{\mathrm{FFT}}$ (here we set $n_s = \frac{N_{\mathrm{FFT}}}{2}$). According to the central limit theorem and the law of large numbers, we know that when we apply different signal-absent observations ($r(n) = w(n)$) for λ times (λ is large enough), the JB statistic values in these different experiments will approximately satisfy a Gaussian distribution with a mean around $0.0344 N_{\mathrm{FFT}}$. That is, the distribution of these JB statistics will be approximately symmetric with respect to this mean. In addition, according to Eq. (7), the JB statistic is non-negative. It means that the smallest possible JB statistic value can only be zero, so subject to the symmetric property we can conclude that most (over 97% of the total population) of the JB statistic values will be smaller than twice of the mean $0.0344 N_{\mathrm{FFT}}$. On the other hand, if there is signal, $R_{\mathrm{out}}(k)$, $k = 0, 1, \ldots, \frac{N_{\mathrm{FFT}}}{2} - 1$ will not satisfy a Gaussian distribution. Thus, the skewness and the kurtosis of $|R_{\mathrm{out}}(k)|$, $k = 0, 1, \ldots, \frac{N_{\mathrm{FFT}}}{2} - 1$ would become larger. According to the aforementioned analysis, we set the threshold r_s for our JB-statistic based detector as

$$r_s = 0.0688 N_{\mathrm{FFT}}. \tag{13}$$

For instance, when we select $N_{\mathrm{FFT}} = 2,048$, which is the defaulted FFT window size according to the DVB standards, the threshold will be $r_s = 141$. Figure 2 depicts the histogram of the JB statistics for a complex Gaussian process over 1,000 random experiments. It can be clearly seen that all JB statistic values in Figure 2 are below the threshold $r_s = 141$.

IV. Simulation

In our simulation, we test two types of commonly-used signals, namely DTV signal and microphone signal to benchmark the spectrum sensing methods. The simulation details are stated as follows.

A. Signal Acquisition and System Set-up

Subject to the IEEE 802.22 standard, the recorded DTV channels were sampled at 21.524476 Msamples/sec and then down-converted to a low central IF frequency of 5.381119 MHz (a fourth of the sampling rate). The real DTV data were acquired from [6]. On the other hand, according to [10], we simulate the microphone signal $s_{\mathrm{mic}}(t)$ as follows:

$$s_{\mathrm{mic}}(t) = \cos\left(2\pi\int_0^t \left[f_{cm} + f_{\triangle}\, w_m(\tau)\right] d\tau\right), \tag{14}$$

where f_{cm} is the same frequency as that of the DTV pilots; $f_{\triangle}$ is the frequency deviation around 100 KHz; $w_m(\tau)$ is the source signal which is randomly generated from the uniformly-distributed number in (-1,1). In addition, the sampling frequency for $s_{\mathrm{mic}}(t)$ is 21.524476 MHz, which is the same as that of the captured DTV signal.

According to [11], the receiver noise characteristic consists of a typical noise power spectral density (PSD) and a noise uncertainty. The noise uncertainty specification is necessary since even though the sensing mechanism may involve calibration based on the noise power estimation, the estimate often exhibits some inaccuracy, which must be modeled. The thermal noise PSD is $N_0 = -174$ dBm/Hz. The receiver noise level is larger than the thermal noise level. Considering the effects of low-noise amplifier (LNA) noise figure, coupling losses, radio frequency (RF) switch losses and other issues, the TV industry typically specifies a *composite receiver noise figure* of 11 dB. Hence the average receiver noise PSD is $\bar{N} = N_0 + 11 = -163$ dBm/Hz.

Moreover, according to the IEEE 802.22 document [8], for the purpose of employing the captured signal to evaluate different detection schemes, it is necessary to initially process the captured ATSC-DTV signals. In particular, the SNR can be precisely controlled in the same way by using this initial process for all different spectrum sensing methods. Quoted from [8], the specific steps for the initial process are given as follows.

Step 1): Read an appropriate number of samples from one of the DTV signal files.

Step 2): Filter the signal using a passband filter with a 6 MHz bandwidth and a center frequency of 5.38119 MHz. The filter shall be a "brick wall" filter (i.e. it shall have a flat frequency response with unity gain) which can allow some rare exceptions.

Step 3): Measure the power in the received signal.

Step 4): Generate white noise sampled at 21.524476 MHz and filter it through the same filter used in Step 2. The noise power used is the receiver noise power.

Step 5): Scale the signal power to meet the target SNR.

Step 6): Add the filtered noise with the scaled and filtered signal.

B. Spectrum Sensing Performance Comparison

In the following, we will present the simulation results for comparing our JB-statistic based detector and the HOS detector. First, the wireless microphone signals according to [6] (randomly generated from computer) and the captured DTV signals from [6] (from the real world) are generated for the benchmark. In the simulation, we set $N_{\mathrm{FFT}} = 2048$, which is also used in the OFDM modulator/demodulator.

In Figure 2, we set the sample size N as 150,000 (almost the minimum required sample size for almost all existing spectrum sensing techniques [1], [2], [6], [7]) and depict the histogram of the JB statistic values from 1000 random experiments. In Figure 3, we delineate the false detection rates resulting from the HOS detector and our JB-statistic based detector versus the sample size N in the sole presence of AWGN. According to Figure 3, it is obvious that when the sample size is larger than 50,000, both our JB-statistic based detector and the HOS detector have very low false detection rates. As the sample size gets smaller (< 50000), in other words, when the sensing time is short, the HOS detector leads to an extremely high false detection rate. Nevertheless, our proposed JB-statistic based detector can still work very well. In Figure 4, we depict the detection rates for simulated wireless microphone signals over 1000 Monte Carlo experiments with $N = 150000$. In Figure 5, we plot the detection rates for real DTV signals over 1000 Monte Carlo experiments with $N = 150000$. According to Figures 4 and 5, our JB-statistic based detector always outperforms the HOS detector across different signal-to-noise ratios in terms of detection rate.

V. Conclusion

In this paper, we propose a novel JB-statistic based spectrum sensing method, which can be applied for the IEEE 802.22 systems. Our method outperforms the existing HOS detection scheme which is broadly adopted in practice. According to our Monte Carlo simulation results for the simulated wireless microphone signals and the real DTV signals, our proposed JB detection method leads to a higher detection rate than the HOS detector. Besides, our proposed JB-statistic based detector can be very robust for small sample size or short sensing time.

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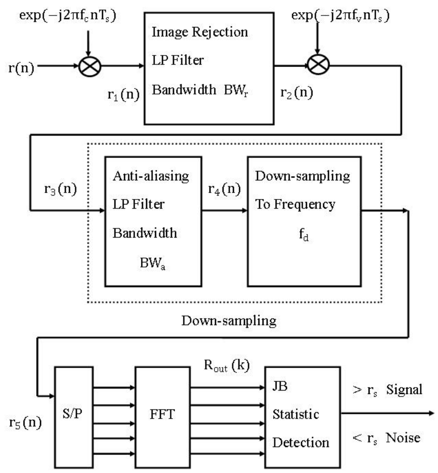

Fig. 1. The spectrum sensing system diargam.

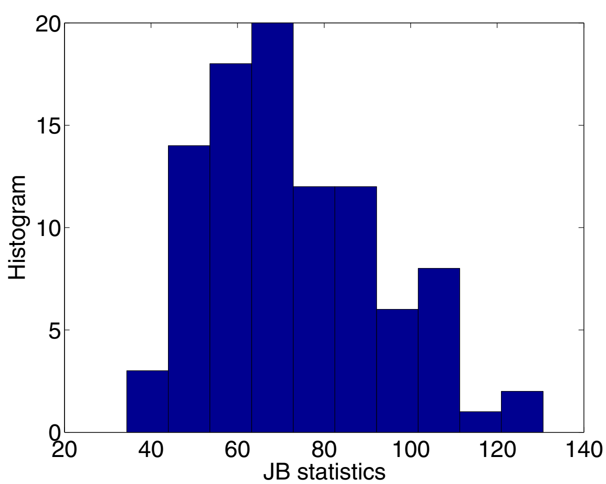

Fig. 2. A histogram example of the JB statistics.

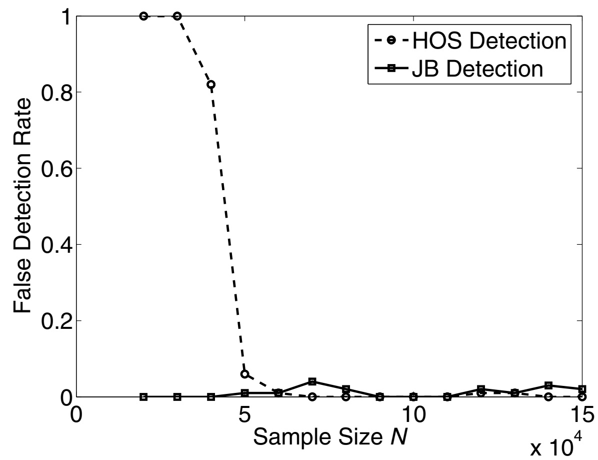

Fig. 3. False detection rate versus sample size in the sole presence of AWGN.

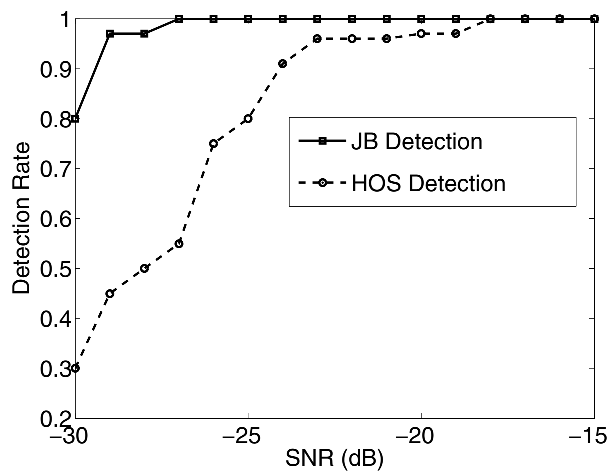

Fig. 4. Detection rate for simulated wireless microphone signals versus SNR ($N = 150,000$).

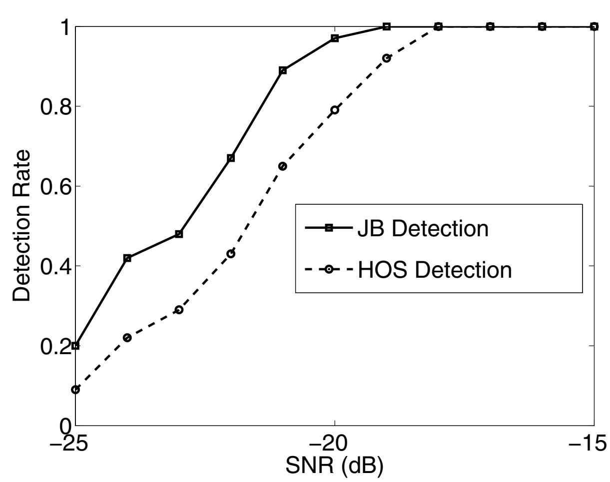

Fig. 5. Detection rate for real DTV signals versus SNR ($N = 150,000$).