Fault-Tolerant Integration of Abstract Sensor Estimates Using Multiresolution Decomposition[1]

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Abstract **This paper proposes a method of applying the idea of multiresolution to the problem of fault-tolerant integration of abstract sensor estimates when the number of sensors is very large and a large number of sensor faults are tame. We give an optimal $O(N\log N)$ algorithm, where N is the total number of sensors, which implements this idea efficiently.**

key words and phrases: ***multiresolution decomposition, fault-tolerant sensor integration***

I. INTRODUCTION

The distributed sensor processing problem, in the context of distributed sensor networks, involves the problem of fault-tolerant integration of information from multiple sensors. Techniques of fault-tolerant sensor integration have to be robust in the sense that even if some of the sensors are faulty, the integrated output should still be reliable.

We have proposed in this paper a new method of sensor integration using techniques of multiresolution decomposition. Multiresolution decomposition is an image decomposition in frequency channels of constant bandwidth on a logarithmic scale. Multiresolution transforms have been the focus of extensive study after the work on multiscale edge detection by Rosenfeld and Thurston[1]. The details of an image characterize different types of physical features at different scales. While at a coarse resolution one can distinguish the gross shapes of the large objects in an image, the exact contours, textures, and smaller details can be discerned at successively finer resolutions. Multiresolution representations offer a simple hierarchical framework for interpreting image information. The approximation of a signal f at a resolution r is deflned as an estimate of f derived by uniformly sampling f, r times per unit length. Tanimoto and Pavlidis[2] have developed efficient algorithms to compute the approximation of a function at different resolutions.

In this paper, we propose a method of applying the idea of multiresolution to the problem of fault-tolerant integration of abstract sensor estimates when the number of sensors is very large, and a large number of sensor faults are tame. The idea essentially consists of constructing a simple function from the outputs of sensors in a cluster and resolving this function at various successively finer scales to isolate the region over which the correct sensors lie. We give an optimal algorithm which implements this idea efficiently.

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A Distributed Sensor Network consists of spatially distributed sensors that detect and quantify a certain phenomenon via its changing parameters. The readings of sensors are sent at regular intervals of time to processing units that integrate these readings and give outputs whose nature is much the same as the inputs of the sensors. Output from processors representing clusters of sensors are later integrated to get a complete picture of the spatially distributed phenomenon. However, before integration is performed at the processor level, it is necessary to have reliable estimates at each processor. Each sensor in a cluster measures the same parameter. It is possible that some of these sensors are faulty. Hence it is desirable to make use of this redundancy of the readings in the cluster to obtain a correct estimate of the parameters being read. In short, a fault-tolerant technique of sensor integration to obtain the correct estimate is sought. Marzullo[3] has addressed the problem of fault-tolerant integration of abstract interval estimates and has generalized his estimates to multidimensional sensors[4].

In order to obtain a method of fault-tolerant sensor integration for dynamic real-time applications, we analyze a function called the *Overlap function* introduced in [5], using the techniques of multiresolution decomposition. This approach will be sketched for abstract interval estimates for the sake of clarity. However, the proposed methodology generalizes easily and fruitfully to higher dimensional sensor outputs.

II. BACKGROUND

We recapitulate some of the definitions and notations of an earlier paper[5] that are relevant here:

Definition 1: An *Abstract Sensor* is a sensor that reads a physical parameter and gives out an abstract interval-estimate I_s which is a bounded and connected subset of the real line R.

Definition 2: A *Correct Sensor* is an abstract sensor where the interval estimate contains the actual value of the parameter being measured. If the interval estimate does not contain the actual value of the parameter being measured, it is called a *Faulty sensor*.

Definition 3: Let sensors $S_1, \ldots, S_n$ feed into a processor P. Let the abstract interval estimate of S_j be I_j; $(1 \le j \le n)$, the closed interval $[a_i, b_i]$ with endpoints a_i and b_i. Define the *Characteristic Function* χ_j of the j^{th} sensor S_j, $1 \le j \le n$ as follows:

$$\chi_j(x) = \begin{cases} 1 & \forall\, x \in I_j \\ 0 & \forall\, x \notin I_j \end{cases} \quad \forall\, 1 \le j \le n$$

Definition 4: Let $O(x) = \sum_{j=1}^{N} \chi_j(x)$ be the "overlap function" of the n abstract sensors. For each $x \in \mathbf{R}$, $O(x)$ gives the number of sensor intervals in which x lies, or, the number of intervals overlapping at the point x.

Definition 5: A sensor is *tamely faulty* if it is a faulty sensor and if it overlaps with a correct sensor.

A. Some comments on tame faults and previous work

If f among n sensors are faulty, then by taking $(n-f)$ intersections of the n sensors' interval estimates we are assured that the correct value of the parameter lies in one of these $(n-f)$ intersections. When the number of sensors is large and the number of faults cannot be strictly bounded, the $(n-f)$ intersections may tend to be scattered wildly over the real line, giving poor output estimates. In order to improve the output estimate in these cases, we must be able to further evaluate the $(n-f)$ intersections to choose the "best possible" intersection which contains the correct value with high reliability.

In the method proposed here we assume as before that the number of sensors is very large, that most faults are tame, and we assume no bound on the number of faults. In fact, we let the number of faults vary with the sampling of the sensors' readings.

As the sensors are sampled synchronously at various time intervals, we order the sensors *a priori* by labeling them, dynamically maintain their overlap function $O(x)$, and analyze it at various scales to obtain successively smaller regions which contain the correct value of the parameter observed.

In our earlier papers ([5] and [6) we evaluated each $(n-f)$ intersection by adding up the popularities of all the intervals participating in that $(n-f)$ intersection and using this as the measure of that $(n-f)$ intersection. The popularity of each interval estimate is the number of intervals overlapping with it. The rationale behind this evaluation rests upon the observation that the tamely faulty sensors would lie close to the correct value, thereby overlapping with the correct sensors of an $(n-f)$ intersection, contributing to its reliability. Wild faults would, on the other hand, be uncorrelated to any value on the real line and therefore would spread randomly on the real line and contribute to "noise," and form smaller, less prominent clusters.

The computation of the reliabilities of the $(n-f)$ intersections is computationally arduous and limits real-time applications. However, the pattern of scattering of tamely faulty intervals can be observed by studying the overlap function $O(x)$ at different scales. Regions where $O(x)$ is maximal are of interest, since these are contained in $(n-f)$ intersections for every f. Hence the correct value lies in these regions with very high probability. However there may be several spatially separated maximal intersections, some of them caused by intersections of wildly faulty sensors. In this case we have to look into the spreads of the crests containing maximal intersections. A large spread indicates heavy clustering of sensors, pointing to tame faults, and hence probably the "correct" maximal intersection. However this analysis of simultaneously selecting crests with large amplitudes and wide spreads cannot be performed on $O(x)$ directly. $O(x)$ has features over several scales. We observe $O(x)$ from a coarse scale and select a crest with largest amplitude and widest spread. We then "zoom-in" on this crest by increasing the resolution of observation and again perform a selection based on largest amplitude and widest spread. This operation is performed over a series of scales, from coarse to fine, resulting in the isolation of a small connected region of the real line over which $O(x)$ takes the highest values.

III. MAIN IDEA

Given a sequence of increasing resolutions $\{r_j\}_{j \in z}$ the details of a function $f(x)$ at the resolution r_j are defined as the difference of information between the approximation of $f(x)$ at the resolution r_{j+1} and the approximation at the resolution r_j.

At a coarse resolution the image details are characterized by very few samples. Hence, the coarse information processing can be performed quickly. The finer details are characterized by more samples, but the prior information derived from the context, construction and thus speeds up the computation. With a coarse-to-fine strategy, a minimum amount of details necessary to perform a recognition task is processed.

Starting at the coarsest resolution, we select those crests with the highest peaks and choose from these crests the one with the widest spread. At the next highest resolution this crest is again inspected for crests within it with highest amplitudes and among these crests the one with the widest spread is retained for finer analysis at the next resolution. This procedure results in isolating those regions of the real line over which $O(x)$ has a maximum value, corresponding to high overlap degree. By our model this indicates that the isolated region has high probability of containing the correct value of the parameters observed. A diagrammatic demonstration of this procedure is illustrated in Fig's 1 to 5, where $O(x)$ in Fig. 1 is processed using multiresolution decomposition at dyadic scales of resolution. The advantage of this procedure is significant from the point of view of computational speeds, for the coarse-to-fine processing leads to elimination of large regions of the support of $O(x)$ at each resolution. We rigorously formulate this heuristic in order to obtain a real time algorithm which dynamically maintains $O(x)$ and obtains the narrowed output estimate. The maintenance of $O(x)$ requires $O(n\log n)$ time where n is the number of sensors.

IV. ESTIMATES ON SENSOR CLUSTERS

In our model of abstract sensors we assume that i) A large number of sensor faults are tame, and that ii) the length of each interval estimate is bounded below by l and above by L, where $l<L$ and l, L are positive real numbers. (Indeed, a very large interval estimate is too inaccurate to be of any value hence may be discarded. On the other hand, a very small interval estimate would not be amenable for fault-tolerance analysis. A minimum tolerance of $\pm l/2$ is built into the abstract sensors, and so we may assume that the width of each interval is at least l). These two assumptions imply that the tame faults cluster in a bounded neighborhood of the correct value of the measured parameter. When the number of

faulty sensors are significant, since most faults are tame, this results in overlaps of the faulty sensors amongst themselves and boosts the value of $O(x)$ in the neighborhood of the correct value of the parameter, thus reinforcing the $(N-f)$ intersection containing the correct value.

Indeed, let T be the number of tamely faulty sensors. These may range in width from l to L. A tamely faulty sensor must intersect with a correct sensor. Therefore its endpoint nearest to the correct value c must lie within a distance of almost L from c. Thus at most $(1+\lfloor L/l \rfloor)$ tamely faulty sensors can be accommodated in either side of c with no two of them overlapping. i.e., at least $\lceil T/2(1+\lfloor L/l \rfloor) \rceil$ tamely faulty sensors overlap over a region of width at least l within a distance of at most $2L$ from c.

When the number of intersections of tamely faulty sensors is $\lceil T/2(1+\lfloor L/l \rfloor) \rceil$, the width of this intersection is actually at least $2(l+L)$. When the number of intersections is T then this results in a peak with spread of at least l. This clustering reinforces the width and height of the 'correct' $(N-f)$ intersection by adding in its neighborhood a peak of area Tl at least. In general this results in a taller and wider peak in the neighborhood of c. The widely faulty sensors are on the other hand random in their location on the real line and being uncorrelated, tend not to cluster around in any restricted neighborhood. Thus the $(N-f)$ intersections resulting from them have shorter and narrower peaks representing them in $O(x)$. We use multiresolution to isolate the robust peaks in $O(x)$ by looking at $O(x)$ at a coarse resolution, and then refine $O(x)$ over the regions over which the robust peaks occur by successive finer resolutions and at each stage reduce the region to finally obtain a narrowed interval in which c is most likely contained. We develop the method in sections to follow.

V. MULTIRESOLUTION OF THE OVERLAP FUNCTION

If S_i $(1 \le i \le N)$ are N abstract sensors with their interval estimates $[a_i, b_i]$ $(1 \le i \le N)$ having characteristic function c_i $(1 \le i \le N)$ then the overlap function $O(x)$ of these N sensors is given by

$$O(x) = \sum_{i=1}^{N} \chi_i(x)$$

For each j, $O(x)$ can be sampled at regular intervals $1/2^j$ to obtain the j-th resolution of $O(x)$ at scale $1/2^j$ as a linear combination of a set of functions obtained by scaling and translating a single function:

Let $\sigma(x) = \begin{cases} 1 & if \quad 0 \le x < 1 \\ 0 & otherwise \end{cases}$. Consider the functions $\{\sigma(2^j x - n)\}_{n=-\infty}^{\infty}$, $j \in \mathbf{Z}$. Explicitly, we have that

$$\sigma(2^j x - n) = \begin{cases} 1 & if \quad n2^j \le x < (n+1)2^{-j} \\ 0 & otherwise \end{cases}$$

The j^{th} resolution of $O(x)$ with respect to the system $\{\sigma(2^j x - n)\}_{n=-\infty}^{\infty}$ is given by:

$$O^j(x) = \sum_{n=-\infty}^{\infty} O(n2^{-j})\sigma(2^j x - n)$$

Since $O(x)$ has compact support, the above summation is actually over finitely many n. Indeed, if $a = \min_{1 \le i \le n}\{a_i\}$ and $b = \min_{1 \le i \le n}\{b_i\}$, then $O^j(x) = \sum_{\lceil 2^j a \rceil}^{\lceil 2^j b \rceil} O(n2^{-j})\sigma(2^j x - n)$.

Thus $O^j(x)$ is obtained from $O(x)$ by sampling at the points $\{n2^{-j}\}_{n=-\infty}^{\infty}$ lying between a and b. $O^j(x)$ is a function whose features are of 'size' 2^{-j} or greater. To study the effect of sampling on this function's representation at any given resolution, it is sufficient to consider the effect of sampling on the test function $g(x) = \begin{cases} 1 & if \quad x \in [\alpha, \beta] \\ 0 & otherwise \end{cases}$, where $[\alpha, \beta]$ is an arbitrary closed interval.

$g^j(x) = \sum_{\lceil 2^j \alpha \rceil}^{\lceil 2^j \beta \rceil} g(n2^{-j})\sigma(2^j x - n)$ is a function which takes the constant value 1 over a semi-closed interval. There are two things which may happen independently to the support of $g^j(x)$ depending upon the incidence of the sampling points: i) It may be shorter on the left by a length of at most 2^{-j} than $[\alpha, \beta]$ ii) It may be longer on the right by a length of at most 2^{-j} than $[\alpha, \beta]$.

We will see later that we have to correct for a positive shrinkage of the support of a feature, so as not to lose any information (correct value of the parameter measured). This is done by resolving over a region bigger than the one at hand by $1/2^j$ on the left. The extension ('Smearing') of support will decrease with further resolution, and does not pose a problem.

While considering resolutions of $O(x)$, we have to choose the scale appropriately. Too large a scale will provide no useful information about the structure of O, while too small a scale would not isolate the features important to us, by bringing in unnecessary detail. And since each sensor has width at least l, it is desirable to start off with a scale smaller than l (or the same order as l) i.e., choose $j = log 1/l$. Thus each sensor will figure as a feature at least as big as $1/2^j$.

The fluctuations in $O(x)$ occur at the points a_i, b_i $(1 \le i \le N)$ which are the end points of the interval estimates. If a is the least of the a_i and b is the largest of the b_i, then the average number of fluctuations per unit length is given by $2N/(b-a)$. So in order to capture all the fluctuations we would have to resolve at least to a level $j > log\,(2N/(b-a))$.

VI. SELECTION OF ROBUST PEAKS

At the j-th level of resolution O^j can be looked upon as a series of juxtaposed peaks. In other words, consider the sequence $\{O(n/2^j)\}$. This sequence is a concatenation of several bitonic sequence, each of which increases first and then decreases--(a bitonic sequence is a sequence of numbers $a_0, \ldots, a_{n-1}$ with the property that (1) there exists an index i, $0 \le i \le n-1$, such that a_0 through a_i is monotonically increasing and a_i through a_{n-1} is monotonically decreasing or else (2) there exists a cyclic shift so that the first condition is satisfied.

For more details see Michael J. Quinn[1987] "Designing Efficient Algorithm for Parallel Computers," McGraw-Hill publication.) Each bitonic sequence which increases first and then decreases corresponds to a peak in O^j. We wish to isolate those peaks which are the tallest and have the widest spread, for it is in the region over which these peaks lie that the correct value of the parameter being measured is most likely to be found. Since the characteristic function of each sensor adds an area numerically equal to the sensor's width to the area under $O(x)$, a good measure of the peaks' robustness is the area under them.

At the j-th resolution consider the sequence $\{O(n/2^j)\}_n$. This is a finite sequence since the support of O is finite. Let there be p peaks (or p bitonic sequences that increase first a then decrease) in O^j. Thus the sequence $\{O(n/2^j)\}$ can be rewritten as

$\{O(n_0/2^j), O((n_0+1)/2^j), \ldots, O(n_1/2^j), O((n_1+1)/2^j), \ldots, O(n_2/2^j), \ldots, O((n_{p-1}+1)/2^j), \ldots, O(n_p/2^j)\}$,

where the subsequence $\{O((n_{k-1}+1)/2^j), \ldots, O(n_k/2^j)\}$ is the k-th bitonic sequence from the left. Therefore the area under this peak is given by

$$1/2^j \sum_{n=n_{k-1}}^{n_k} O(n/2^j)$$

Since the factor $1/2^j$ is common to the areas of all peaks at the j-th resolution. We may make the area 'scale-free' by dropping this factor and writing the area of the k-th peak at level j as

$$\mathbf{A}^j(k) = \sum_{n=n_{k-1}}^{n_k} O(n/2^j)$$

We then select the peaks with largest area and ignore the other peaks. The function O is further resolved over the regions over which these largest peaks occur, and the process is repeated until a satisfactory region of the real line is isolated as the most likely candidate for containing the correct value of the parameter being measured by the sensors. However, before resolving a certain selected peak further, we correct the region over which the resolution is to be carried out by adding a segment of length $1/2^j$. Fig's 1 through 5 show the property of coarse grain to fine grain scheme for isolating robust peaks.

Indeed if at the j-th resolution the k-th peak is selected as the peak with the largest area under it, then the region over which the resolution of $O(x)$ is performed again is $[(n_{k-1}+1)/2^j, n_k/2^j]$ with a correction of length $1/2^j$ to the left.

Therefore $O(x)$ is resolved over $[n_{k-1}/2^j, n_k/2^j]$. The process of resolving and selecting peaks with largest area is continued until the width over which further resolution is to be carried out is smaller than the maximum acceptable width for the integrated output estimate, or when further resolution does not reduce the region selected from the previous resolution. The final corrected region with the largest value of O over it is accepted as the output estimate. We will now discuss the algorithmic aspect this procedure below.

VII. THE ALGORITHM

In this section, we give an algorithm which implements our analytical method.

Input. The end points a_i, b_i of the interval estimate $[a_i, b_i]$ of the sensors S_i, " $1 \le i \le N$. The lower and upper bounds of resolution j_0 and j_1 ($j_0 = log(1/l)$ and $j_1 > log(2N/Supp(O(x)))$.

begin

1. Form the array of ordered pairs: $[(a_1, 1), (b_1,-1), (a_2, 1), (b_2,-1), \ldots, (a_N, 1), (b_N,-1)]$.
2. Sort this array in increasing order with respect to the first components of the ordered pairs to obtain the array $[(\alpha_1, \sigma_1), (\alpha_2, \sigma_2), \ldots, (\alpha_{2N}, \sigma_{2N})]$, where each α_i is some a_j or b_j, $\alpha_i \le \alpha_{i+1}$ $1 \le i \le 2N$, and
$$\sigma_i = \begin{cases} 1 & \text{if } \alpha_i \text{ is an } a_j \\ -1 & \text{if } \alpha_i \text{ is a } b_j \end{cases}$$
3. Construct the array $[(-\infty,0), (\alpha_1,\sigma_1), \ldots, (\alpha_i, \sum_{j=1}^{i}\sigma_j), \ldots, (\alpha_{2N}, \sum_{j=1}^{2N}\sigma_j), (\infty,0)]$ representing the overlap function $O(x)$.

(note that $\sigma_1 = +1$, $\sum_{j=1}^{2N}\sigma_j = 0$, and $\sum_{j=1}^{i}\sigma_j$ $\alpha_i \le x \le \alpha_{i+1}$, $0 \le i \le 2N$ where $\alpha_0 = -\infty$ and $\alpha_{2N+1} = \infty$).

Set $n_{j_0} = \lfloor 2^{j_0}\alpha_1 \rfloor$ .and $n'_{j_0} = \lceil 2^{j_0}\alpha_{2N} \rceil$

while $j_0 \le j < j_1$ **do**

RESOLVE[j; n_{j-1}, n'_{j-1}]

$j := j + 1$

end while

Sample $O(x)$ between $n_{j_1}/2^{j_1}$ and $n'_{j_1}/2^{j_1}$ to obtain $O^{j_1}(x)$: the approximation of $O(x)$ at the j_1 th resolution. Choose that subinterval of $[n_{j_1}/2^{j_1}, n'_{j_1}/2^{j_1}]$ over which $O^{j_1}(x)$ attains a maximum (or takes values greater than a specified value) and accept this subinterval as the integrated output estimate of the N sensor estimates.

end.

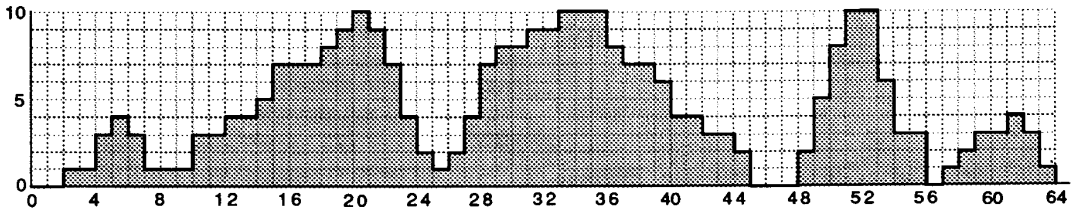

Fig 1: $O(x)$, the shaded region indicates the portion which is to be resolved over.

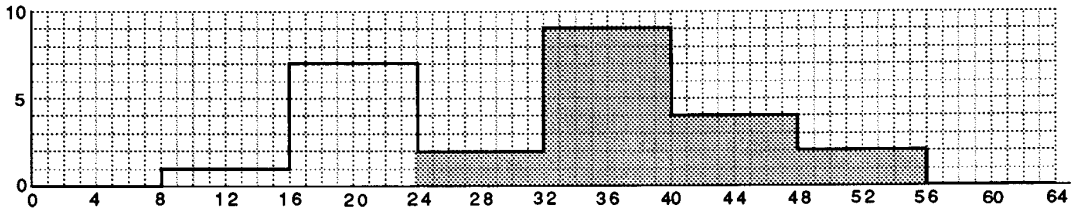

Fig 2: $O^{-3}(x)$, the shaded region and the region of width 8 to its left are to be resolved over.

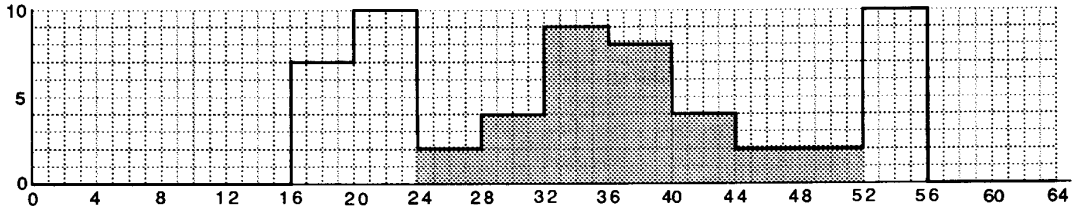

Fig 3: $O^{-2}(x)$, the shaded region and the region of width 4 to its left are to be resolved over.

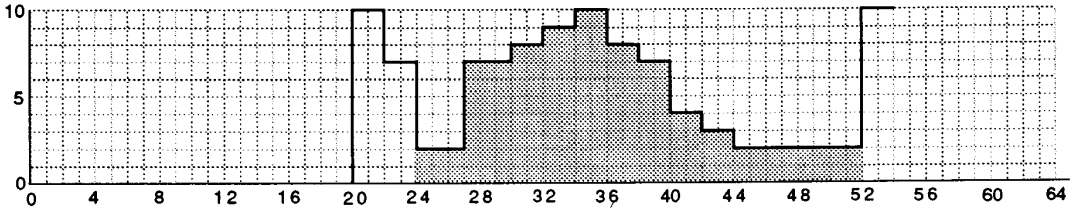

Fig 4: $O^{-1}(x)$, the shaded region and the region of width 2 to its left are to be resolved over.

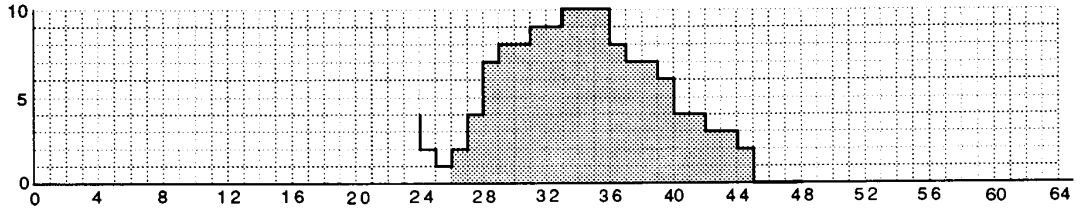

Fig 5: $O^{0}(x)$, the shaded region and the region of width 1 to its left are to be resolved over. We may at this point terminate resolution and choose the interval over which this peak attains a maximum.

The subroutine RESOLVE (below) which resolves $O(x)$ to obtain an approximation of O with the j-th resolution over a given interval is given below. It yields the indices of two points at the j-th resolution, over which the largest or most prominent peak at the j-th resolution occurs.

RESOLVE[j; n_{j-1}, n'_{j-1}]
begin

1. Resolve $O(x)$ at scale 2^{-j} over the interval $[(n_{j-1}-1)/2^{j-1},\ n'_{j-1}/2^{j-1}]$ by sampling $O(x)$ at the points $(2n_{j-1}-2)/2^j$, $(2n_{j-1}-1)/2^j$, ... , $2n'_{j-1}/2^j$ to obtain $O^j(x)$: the approximation of $O(x)$ at the j-th resolution, represented by the array $[(-\infty, 0), \ldots, (n/2^j, \sum_{j=1}^{k_n} \sigma_j), \ldots, (\infty, 0)]$ $(2n_{j-1} - 2 \le n \le 2n'_{j-1})$ and $O^j(x) = \sum_{j=1}^{k_n} \sigma_j$ " $n/2^j \le x \le (n+1)/2^j$, where $(n/2^j - \alpha_{k_n}) < 1/2^j$.
2. Choose n_j and n'_j, where $n_j < n'_j$ and $2n_{j-1} - 2 \le n_j \le 2n'_{j-1}$ such that $\left\{O\left(n/2^j\right)\right\}_{n=n_j}^{n'_j}$ is a contiguous bitonic subsequence of $\left\{O\left(n/2^j\right)\right\}_{n=2n_{j-1}-2}^{2n'_{j-1}}$ which first increases and then decreases, and which has the largest sum. i.e., $\sum_{n=n_j}^{n'_j} O\left(n/2^j\right)$ is maximum of all such bitonic subsequences' sum.

end.

Fig's 1 to 5 depict the implementation of this algorithm graphically.

Complexity of the Algorithm

The subroutine RESOLVE involves only scanning and hence is linear in the number of sensors N. Since the average density of fluctuation in O is $2N/Supp(O(x))$, the level of resolution j required to capture almost all the measures of O is given by $j > log(2N/Supp(O(x)))$.

If we assume that the parameter being measured by the sensors is known to lie between certain bounds, then $j > log\ N + C$ where C is some constant. Thus the subroutine RESOLVE will be called on an average $O(logN)$ times. Hence the average complexity of the algorithm is $O(NlogN)$.

CONCLUDING REMARKS

In this paper we have applied a novel technique to process the inputs of several sensors to obtain an accurate and fault-tolerant interval estimate as the integrated output. The idea behind this application is the recognition and isolation of the most prominent and robust peaks in a region and the consequent elimination of narrower and less prominent peaks as 'errors.' This application of multiresolution can indeed be used elsewhere to recognize the important characteristics of a signal and overlook 'noise' factors in a computationally efficient manner. This method can be generalized with some modifications to multidimensional sensors and signals. The above method and algorithm can be reformulated using Haar wavelets and the fast wavelet algorithm also.

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