• Please remember that all submissions must be typeset. Handwritten submissions will NOT be accepted. These must be uploaded to SCIS moodle in PDF format only.

• Please remember to type your name on top of your submission.

1. (20 points) We learned in class the hare-tortoise algorithm of cycle-finding in a linked list. Based on this, design an algorithm that is both time and space efficient which finds the starting node of the cycle.

2. (20 points) A patient has \( n \) pills to take. In each day, he can take either one pill or two pills until all pills are gone. Let \( T(n) \) denote the number of different ways the patient can take all \( n \) pills. Give a closed form for \( T(n) \). (Note that – for example – the two sequences (1, 2, 2) and (2, 1, 2) are considered as two different ways of taking 5 pills.)

3. (20 points) An array \( A \) of \( n \) distinct numbers are said to be unimodal if there exists an index \( k \), \( 1 \leq k \leq n \), such that \( A[1] < A[2] < \cdots < A[k-1] < A[k] \) and \( A[k] > A[k+1] > \cdots > A[n] \). In other words, \( A \) has a unique peak and are monotone on both sides of the peak. Design an efficient algorithm that finds the peak in a unimodal array of size \( n \).

4. (20 points) Suppose we are given two sorted arrays \( A \) and \( B \) each consisting of \( n \) numbers, and an integer \( 1 \leq k \leq 2n \). Describe an algorithm that finds the \( k^{th} \) smallest element in the union of \( A \) and \( B \) in \( O(\log n) \) time.

5. (20 points) Your friend claims that it is asymptotically faster to square an \( n \)-bit integer than to multiply two \( n \)-bit integers. Should you believe your friend? Why?