COP-4534: Algorithm Techniques

Homework 5

DUE: Wednesday December 6 at 11:55 PM

- Please remember that all submissions must be typeset. Handwritten submissions will NOT be accepted. These must be uploaded to SCIS moodle in PDF format only.

- Please remember to type your name on top of your submission.

1. (20 points) You have \( n \) pairs of nuts and bolts (the bolts screw into the nuts) that have been dropped onto a table with the nuts on one side and the bolts on another side; however you cannot tell whether one nut is bigger than another nut or whether one bolt is bigger than another bolt. You can attempt to screw a bolt into a nut and this will tell you either that the nut matches the bolt, or that the bolt is too large or too small for the nut. Call this a nut/bolt comparison. Give a randomized algorithm that will match all nuts with their corresponding bolts with expected \( O(n \log n) \) number of nut/bolt comparisons. (Hint: does this problem look similar to QuickSort?)

2. (20 points) In the following, assume that all keys are distinct.

   (a) Consider a hash table with \( m \) buckets that uses chaining for collision resolution. The table is initially empty. What is the probability that, after \( k \) keys are inserted, there is a chain of size \( k - 1 \)?

   (b) Consider a hash table with \( m \) slots that uses open addressing with linear probing. The table is initially empty. A key \( k_1 \) is inserted into the table, followed by key \( k_2 \). What is the probability that inserting key \( k_3 \) requires three probes?

3. (20 points) Given an array of positive integers \( A[1..n] \), where \( A[i] \) represents the maximum number of steps that can be made forward from position \( i \). Design an efficient algorithm that finds the minimum number of jumps starting from position 1 to reach the end of the array (position \( n \)).

4. (20 points) You are a busy businessman with \( n \) consecutive working days to plan. There are \( m \) businessmen whose available time slots are given: person \( i \) can meet you from day \( s_i \) to day \( t_i \) \((s_i < t_i)\). If you devote your time during this period exclusively with person \( i \), then you will gain a profit of \( v_i \). Help your secretary to plan your days optimally so that your total profit is maximized. What is the running time of your algorithm?

5. (20 points) You are given an ordered sequence of \( n \) cities, as well as the distances between each pair of cities. Your goal is partition the cities into two subsequences (not necessarily contiguous) such that person \( A \) visits all cities in the first subsequence (in order), person \( B \) visits all cities in the second subsequence (in order), and such that the sum of the total distances travelled by \( A \) and \( B \) is minimized. Assume that person \( A \) and person \( B \) start initially at the first city in their respective subsequences.