

Syllabus for MAS 3105 Linear Algebra, Class Number 22963, Spring 2018

Book: *Vectors, Pure and Applied* by T. W. Körner, ISBN-13: 978-1107675223

Synopsis of the course

Linear Algebra is a comparatively young branch of Mathematics (less than 100 years old) which studies the concept of *linearity over a ring* encoded in *modules and morphisms between them*. The theory of these is addressed (in significant detail) in the upper division course MAS 4302 Topics in Algebraic Structures and in the graduate course MAS 5311 Graduate Algebra.

The course MAS 3105 is an undergraduate course aimed to be a very gentle introduction to the area based on thorough study of *modules over the two most important for the applications rings – the fields of real and complex numbers* (modules over fields, a very special type of rings, are called vector spaces). Due to lack of knowledge at this time, the motivation for our considerations will come from solving systems of linear equations rather than from the general set up of modules. The benefit from this approach will be at least two-fold: we will provide the aforementioned courses with extremely valuable examples on one hand, and we will provide the students with basics of powerful techniques used in Applied Math. To this end, we will introduce

- (1) The concept of a matrix, elementary (row- and column-) transformations thereof, the row-echelon form of a matrix. Rank and nullity of a matrix will be introduced.
- (2) The operations addition and multiplication of matrices, and will study when a matrix is invertible. This latter is closely related to the concept of a determinant of a (square) matrix which is carefully explained, too.
- (3) The concept of arithmetic vector space over the real and complex numbers; the matrices are interpreted as morphism between such spaces. This will naturally lead us to the concept of abstract vector space (over the reals or the complex numbers), and to morphisms between them. The concept of basis of a vector space will be introduced and studied. It will turn out that matrices are descriptions of morphisms of vector spaces.
- (4) The concept of an operator of a vector space (these are known also as endomorphisms of a vector space), eigenvalue and eigenvector thereof. The matrices associated with operators are square, the eigenvalues of an operator are roots of the characteristic polynomial of an associated matrix. Among such matrices, there are some of simplest form – diagonal ones. We study conditions under which an operator is diagonalizable, that is, has a diagonal associated matrix.

This material is covered in the first six chapters of Körner's book.

Assessment of students' knowledge: The overall grade will be based on the results on several Quizzes, Turn-in Homework Assignments, two Midterms, and the Final Exam. All the problems for the exams will be taken from the ones given for work at home during the Semester. There is a file with solutions to the embedded in the main text exercises available (on the web site of the author, as well as on the web site of the course). The overall grade will be based on 15% of the Turn-in homework's total score, 15% of the total Quizzes' score, 30% of the

total of the Midterms' scores, and 40% of the Final Exam score.

The scale for the overall grades follows:

Example: Suppose a student has A points total on the HW, B points total on the Quizzes, C points total on the Midterm Exams, and D points on the Final Exam. Suppose further that the maximal possible points one can get on these are A', B', C', and D' respectively. Then, one can compute a number S by the formula

$$S = [15*A + 15*B + 30*C + 40*D] / [15*A' + 15*B' + 30*C' + 40*D'].$$

The overall grade of the student above is determined now by the scale:

$0.92 < S$: A	$0.89 < S < 0.92$: A-	$0.86 < S < 0.89$: B+
$0.78 < S < 0.86$: B	$0.75 < S < 0.78$: B-	$0.71 < S < 0.75$: C+
$0.62 < S < 0.71$: C	$0.58 < S < 0.62$: C-	$0.55 < S < 0.58$: D+
$0.49 < S < 0.55$: D	$0.46 < S < 0.49$: D-	$S < 0.46$: F

Note: No make-up exams will be scheduled.

Important remark: The Instructor reserves the right to make any changes he considers academically advisable. Any such changes will be announced in advanced in class or by posting them to the e-mail accounts of the students. The students are responsible to be aware of the changes announced this way.