COT 3420 — Logic for Computer Science
Homework Assignment 4
Due Monday, March 31

Construct proof tableaux to prove the correctness of each of the following programs. Argue, at least informally, that all verification conditions hold.

1. **Swapping with No Temporary**:

   $\{x = X \wedge y = Y\}$
   $x := x + y;$
   $y := x - y;$
   $x := x - y$
   $\{y = X \wedge x = Y\}$

2. **Maximum**:

   $\{T\}$
   if $x \geq y$ **then**
   $\quad z := x$
   else
   $\quad z := y$
   fi
   $\{z = \mathit{max}(x, y)\}$

3. **Quotient and Remainder**:

   $\{n \geq 0 \wedge d > 0\}$
   $q := 0;$
   $r := n;$
   while $r \geq d$ **do**
   $\quad q := q + 1;$
   $\quad r := r - d$
   od
   $\{n = q * d + r \wedge 0 \leq r < d\}$

   This program computes the quotient q and remainder r when n is divided by d. For your loop invariant I, use

   $$n = q * d + r \wedge r \geq 0 \wedge d > 0.$$

For your bound function t, you can use r. (Or, if you want to be fancier, you can use $\lfloor r/d \rfloor$.)

4. **The Coffee Can Problem**:

   A coffee can contains some black beans and some white beans. Repeatedly, play the following game:

   > Pick two beans at random. If they are of the same color, throw them away, but put another black bean into the can. (Enough extra black beans are available to do this.) If they are of different colors, put the white one back into the can and throw the black one away.

   Each time this is done, the number of beans in the can goes down by one. The game ends when there is only one bean left.

   If you know how many beans of each color are in the can to begin with, can you predict the color of the final bean?