

**COT 3420 — Logic for Computer Science**  
**Homework Assignment 5**  
**Due Monday, April 14**

Develop programs to solve the following problems. Your solutions should describe *how* you derived your programs, as I have done in the class lectures.

1. Write a program that, given integer  $n \geq 1$ , sets  $i$  to the largest power of 2 that is less than or equal to  $n$ . (The powers of 2 are 1, 2, 4, 8, 16, 32, 64, 128, and so on.) For example, if  $n$  is 40, then you should set  $i$  to 32. (Hint: first write down a formal specification and then derive the invariant by deleting a conjunct.)
2. Recall that a *plateau* of an array  $b$  is a nonempty segment  $b[i..j]$  all of whose values are equal. A plateau  $b[i..j]$  is *maximal* if neither  $b[i-1..j]$  nor  $b[i..j+1]$  is a plateau. Write a program to count the number of maximal plateaus in  $b[0..n-1]$ , where  $n \geq 1$ .
3. Given are arrays  $a[0..m-1]$  and  $b[0..n-1]$ , where  $m \geq 0$  and  $n \geq 0$ . Furthermore, both  $a$  and  $b$  are *strictly* ascending. Write a program to determine the number of common values of  $a$  and  $b$ . That is, establish

$$k = (Ni, j : 0 \leq i < m \wedge 0 \leq j < n : a[i] = b[j])$$

Your program must take time proportional to  $m+n$ . (Hint: this problem requires the technique of *strengthening the invariant* to avoid inefficiency. In particular, it is convenient to make use of *imaginary elements*  $a[-1] = b[-1] = -\infty$  and  $a[m] = b[n] = \infty$  in the invariant.)

4. Recall that the code  $x := x + y; y := x - y; x := x - y$  swaps  $x$  and  $y$ , without using a temporary location. Now consider using this approach to try to swap two array elements  $b[i]$  and  $b[j]$ :

$$\begin{aligned} &\{\varphi\} \\ &b[i] := b[i] + b[j]; \\ &b[j] := b[i] - b[j]; \\ &b[i] := b[i] - b[j] \\ &\{b[i] = X \wedge b[j] = Y\} \end{aligned}$$

Complete the tableau, using the assignment axiom for arrays, and determine what precondition  $\varphi$  is needed for the array swap to work correctly.