COT 5420 — Homework 1

Due Monday, September 27

Read Sections 1.1 and 1.2 and solve the problems below. Remember that I am asking you to form a homework group with one or two other students. After you have thought about the problems individually, you should discuss them with your group. Then your group should prepare a single nice write-up of your solutions to submit for grading.

- 1.4
- 1.5
- 1.29 (*Hint*: First solve the case when $n = 5$, then generalize.)
- 1.30 (*Hint*: First solve the case when $n = 5$, then generalize.)
- Recall our discussion of DFAs. Given $\delta : Q \times \Sigma \to Q$, we inductively define $\widehat{\delta} : Q \times \Sigma^* \to Q$ by

  1. $\widehat{\delta}(q, \epsilon) = q$
  2. $\widehat{\delta}(q, ua) = \delta(\widehat{\delta}(q, u), a)$, for $u \in \Sigma^*$, $a \in \Sigma$.

  Prove that for all $x, y \in \Sigma^*$ and $q \in Q$,

  $$\widehat{\delta}(\widehat{\delta}(q, x), y) = \widehat{\delta}(q, xy).$$

  (Hint: use induction on the length of y.)

- We showed in class that swapping the final and non-final states of a DFA M gives a DFA recognizing the complement of $L(M)$. Show that this construction generally fails on NFAs.
- Recall that given an NFA $M = (Q, \Sigma, \delta, I, F)$ we defined NFA M^R by swapping I and F and reversing all the arrows: $M^R = (Q, \Sigma, \delta^{-1}, F, I)$, where $\delta^{-1}(q, a) = \{r \in Q \mid q \in \delta(r, a)\}$. Formally prove that $L(M^R) = L(M)^R$.
- **(Bonus)** Let L be a subset of $\{a\}^*$. We will see later that such an L need *not* be regular; for example $\{a^p \mid p \text{ is prime}\}$ is not regular. It turns out, however, that L^* is always regular. Prove it.