COT 5420 — Homework 2

Due Monday, October 18

Read Sections 1.3 and 1.4 and solve the following problems:

1. Give a regular expression that generates the set of all binary strings containing at least two 0’s and at most one 1.

2. Recall the family of languages $L_n$ discussed in class:

   $$L_n = \{ w \in \{a, b\}^* \mid \text{the } n\text{-th-from-last symbol of } w \text{ is } b \}. $$

   Show that for each $n \geq 1$, $L_n$ can be recognized by an $(n + 1)$-state UFA. Then prove that no DFA with fewer than $2^n$ states can recognize $L_n$.

3. Prove that $\{ a^p \mid p \text{ is prime} \}$ is not a regular language.

4. Prove that $\{ 0^n 1^n \mid m \neq n \}$ is not a regular language.

5. Consider the language $F = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k \}$. Prove that $F$ is not regular. Show that, nevertheless, $F$ has the “pumping property”; that is, there exists $p$ such that for any $w \in F$ with $|w| \geq p$, there exist strings $x, y, \text{ and } z$ where $w = xyz$ and

   (a) for all $i \geq 0$, $xy^iz \in F$,

   (b) $|y| > 0$, and

   (c) $|xy| \leq p$.

6. (Bonus) 1.42