

COT 5420 — Homework 2

Due Monday, October 18

Read Sections 1.3 and 1.4 and solve the following problems:

1. Give a regular expression that generates the set of all binary strings containing at least two 0's and at most one 1.
2. Recall the family of languages L_n discussed in class:

$$L_n = \{w \in \{a, b\}^* \mid \text{the } n\text{th-from-last symbol of } w \text{ is } b\}.$$

Show that for each $n \geq 1$, L_n can be recognized by an $(n + 1)$ -state UFA. Then prove that no DFA with fewer than 2^n states can recognize L_n .

3. Prove that $\{a^p \mid p \text{ is prime}\}$ is not a regular language.
4. Prove that $\{0^m 1^n \mid m \neq n\}$ is not a regular language.
5. Consider the language $F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$. Prove that F is not regular. Show that, nevertheless, F has the “pumping property”; that is, there exists p such that for any $w \in F$ with $|w| \geq p$, there exist strings x , y , and z where $w = xyz$ and
 - (a) for all $i \geq 0$, $xy^i z \in F$,
 - (b) $|y| > 0$, and
 - (c) $|xy| \leq p$.
6. (Bonus) 1.42