

# COT 5420 — Homework 3

Due Monday, November 8

Read Section 2.1 of Sipser and solve the following problems:

1. Sipser problem 2.4
2. Sipser problem 2.8 (Give parse trees rather than leftmost derivations.)
3. Sipser problem 2.15
4. Let  $G$  be the following context-free grammar:

$$S \rightarrow bS \mid Sa \mid aSb \mid \epsilon$$

What well-known language does  $G$  generate? Prove your answer!

5. Let  $L$  be the set of all strings in  $\{a, b\}^*$  that contain twice as many  $a$ 's as  $b$ 's. Here is a grammar  $G$  that attempts to generate  $L$ :

$$\begin{aligned} S &\rightarrow aB \mid bA \mid \epsilon \\ A &\rightarrow aSa \mid aaS \mid baaA \mid baAa \mid bAaa \\ B &\rightarrow baS \mid bSa \mid abS \mid aSb \end{aligned}$$

Prove that  $G$  is *incorrect*.

6. Let  $L$  be the set of all strings in  $\{a, b\}^*$  that are *not* of the form  $ww$  for some  $w \in \{a, b\}^*$ . Thus  $L$  contains  $aba$  and  $abbaab$ , but not  $abbabb$ . Give a CFG that generates  $L$ .

[When I first saw this problem years ago, I found it quite difficult. But it has a very pretty answer!]