COT 5420 — Homework 4

Due Monday, November 22

Read Sections 2.2 and 2.3 of Sipser and solve the following problems:

1. Recall that a CFG is in *Chomsky Normal Form* if every rule is of the form $A \to BC$ or $A \to a$. Show that if G is a CFG in Chomsky Normal Form, and $w \in L(G)$, then any derivation of w has length $2|w| - 1$.

2. Construct a PDA that recognizes

   $$\{w \in \{a, b\}^* \mid w \text{ contains twice as many } a\text{'s as } b\text{'s}\}.$$

3. Prove that the class of context-free languages is closed under intersection with regular languages. That is, prove that if L is a CFL and R is regular, then $L \cap R$ is a CFL. [*Hint*: Give a formal PDA construction.]

4. Prove that

   $$\{w \in \{a, b, c\}^* \mid w \text{ contains an equal number of } a\text{'s, } b\text{'s, and } c\text{'s}\}$$

   is not a CFL. [*Hint*: Use problem 3.]

5. Prove that $\{a^m b^n a^m b^n \mid m, n \geq 0\}$ is not a CFL.

6. Is $\{a^i b^j \mid i \neq j \text{ and } i \neq 2j\}$ a CFL? Prove your answer.

7. (Bonus) Let L be a subset of $\{a\}^*$. Prove that if L is a CFL, then L is regular.