Complementary Operators

It is important to note that each relational operator is a complement of one other relational operator. The operator > has complement <=; similarly < has complement >=; and lastly, == has complement !=. You may be wondering what is the significant of this. It has been known that positive logic is easier to read and understand than negative logic. That is, reading a conditional expression that is preceded by the negation sign (!), is harder to read and interpret, than one that does not involve the negation symbol. For example, consider the following statements:

Original Expression	Simplified Expression
! (x < y)	x >= y
! (x > y)	x <= y
! (x == y)	x != y
! (x != y)	x == y
! (x <= y)	x > y
! (x >= y)	x < y

You will notice that the original conditional expression is harder to read than the simplified version. It is harder to comprehend a sentence which goes something like this:

When translated, the statement reads:

```
award = ! (years < 15 )
```

Most people would understand the statement more readily if it were stated in one of the following two ways:

(a) You will get an award if your years of service is at least 15 years, or
(b) You will get an award if your years of service is 15 years or more.

In either case, the complement of the above relational expression would be:

```
award = (years >= 15 )
```

This form would be more readily understood than the first way of expressing the thought.

Just like how relational operators have complement, the logical operators AND (&&), and OR (||) are complement of their one another, when the logical expression is preceded by the NOT (!) operator.

De Morgan's Law

Augustus De Morgan, an Indian born English Mathematician discovered that a logical expression that is written in positive sense is easier to read and understand than a negative logical expression. That is, if the expression begins with the NOT (!) operator, the expression can be complemented to form a positive expression. De Morgan's Law is as follows:

> When we remove the parentheses in a logical expression that is preceded by the NOT (!) operator, we must apply the not operator to each relational expression, while complementing each of the logical operators – that is, changing AND (&&) to OR (||), and vice versa.

Consider the following logical expression:

!(x && y || ! z), where x, y, and z are relational expressions

According to De Morgan's Law this expression would be equivalent to:

!x || !y && !(!z), which make this equivalent to: !x || !y && z,

Example 5.4 Consider the following statement: It is not true that you must both be older than 18 years and earn less than $1500.00 in order to get benefit.

When translated directly, the statement is read as follows:

! (age > 18 && earning < 1500.0)

According to De Morgan's Law this statement, when complemented would be as follows:

!(age > 18) || !(earning < 1500)

Complementing each relational expression would even make the statement more understandable. That is, the equivalent would read:

age <= 18 || earning >= 1500

Self Check

Simplify each of the following by using De Morgan's Law and relational complementation. Evaluate the original version and the simplified version:

(a) !(25 > 30 && 16 == 12 + 4)
(b) !(25 > 30 || 16 == 12 + 4)
(c) !(3 > 7 && !(2 < 0 || 6 * 2 == 24/2))
(d) !(2 < 7) && !(20 > 50 || 30 – 20 == 10))